Boundedness of the objective function

Combinatorial Optimization – Group K

Class 14

Spring 2023

1. Decide whether the following linear programs are solvable. If yes, then decide whether their objective function is bounded from above on the sets of their solutions.

   (a)
   $\max\{9x_1 + 4x_2 + 3x_3\}$
   subject to
   $5x_1 + x_2 + 4x_3 \leq 7$
   $x_1 + x_2 + 5x_3 \leq 2$
   $x_2 \leq 1$

   (b)
   $\max\{2x_1 + 3x_2 + 4x_3 + 5x_4\}$
   subject to
   $x_1 + 2x_2 + x_3 \leq 5$
   $x_2 + 2x_4 \leq 6$
   $x_1 + x_3 + x_4 \leq 7$
   $2x_2 + 3x_4 \leq 8$

   (c)
   $\max\{7x_1 + 8x_2 + x_3\}$
   subject to
   $x_1 + x_2 - 4x_3 \leq 2$
   $3x_1 + 3x_2 + 2x_3 \leq 5$
   $5x_1 + 6x_2 + 7x_3 \leq 2$
   $x_3 \geq 1$

2. For what values of the parameter p is it true that the objective function of the following linear programs is bounded from the relevant direction on the sets of solutions?

   (a)
   $\max\{x_1 + x_2 + p \cdot x_3\}$
   subject to
   $x_1 - x_4 + x_6 \geq 3$
   $-x_1 - x_2 - x_3 \geq 6$
   $x_2 + x_5 - x_6 \leq 1$
   $x_3 + x_4 - x_5 \leq 2$

   (b)
   $\max\{x_4\}$
   subject to
   $3x_1 + 2x_2 + 4x_3 + 5x_4 \leq 4$
   $2x_1 + x_2 + 3x_3 + 4x_4 \geq 1$
   $x_3 + 2x_4 \leq 1$
   $x_2 - 2x_3 + t \cdot x_4 \geq -2$

   (c)
   $\max\{x_1 - x_2 - x_3 - x_4 + p \cdot x_4\}$
   subject to
   $x_1 - 2x_2 \leq 1$
   $x_2 - 2x_3 \leq 2$
   $x_3 - 2x_4 \leq 3$
   $x_4 - 2x_5 \leq 4$