Complementary Slackness

1. Decide if $x_1 = 4$, $x_2 = 0$, $x_3 = 3$, $x_4 = 0$ is an optimal solution of the following linear programs.

   (a)
   $$\max\{6x_1 + 5x_2 + x_3 + 12x_4\}$$
   subject to
   $$x_1 + 2x_2 + x_3 + 5x_4 \leq 7$$
   $$x_1 + 4x_2 + 6x_3 \leq 24$$
   $$-x_1 + 3x_2 + 2x_3 + 3x_4 \leq 3$$
   $$4x_1 + x_2 - x_3 + 2x_4 \leq 13$$
   $$5x_1 + 3x_2 + x_3 + 3x_4 \leq 25$$

   (b)
   $$\max\{7x_1 + x_3 - x_4\}$$
   subject to
   $$x_1 + 2x_2 + x_3 + 5x_4 \leq 7$$
   $$x_1 + 4x_2 + 6x_3 \leq 24$$
   $$-x_1 + 3x_2 + 2x_3 + 3x_4 \leq 3$$
   $$4x_1 + x_2 - x_3 + 2x_4 \leq 13$$
   $$5x_1 + 3x_2 + x_3 + 3x_4 \leq 25$$

   (c)
   $$\max\{x_1 + 2x_2 + x_3 + 4x_4\}$$
   subject to
   $$x_1 + 2x_2 + x_3 + 5x_4 \leq 7$$
   $$x_1 + 4x_2 + 6x_3 \leq 24$$
   $$-x_1 + 3x_2 + 2x_3 + 3x_4 \leq 3$$
   $$4x_1 + x_2 - x_3 + 2x_4 \leq 13$$
   $$5x_1 + 3x_2 + x_3 + 3x_4 \leq 25$$

2. Decide if $x_1 = 1$, $x_2 = 2$, $x_3 = 1$, $x_4 = 2$ is an optimal solution of the following linear program.

   $$\max\{5x_1 - 10x_3 + x_4\}$$
   subject to
   $$5x_1 - 10x_3 + x_4 \leq -3$$
   $$5x_2 + 2x_4 = 14$$
   $$x_3 + 4x_4 \leq 11$$
   $$2x_1 - 3x_2 - 4x_3 - x_4 \geq -10$$

3. Decide if $y_1 = 3$, $y_2 = 0$, $y_3 = 2$, $y_4 = 1$ is an optimal solution of the following linear program.

$$\min\{7y_1 + 15y_2 + 11y_3 + 5y_4\}$$
subject to
$$y_1 + 2y_2 + 2y_3 + y_4 = 8$$
$$5y_2 = 0$$
$$y_1 + y_3 = 5$$
$$2y_1 - y_2 + 7y_3 + 4y_4 = 24$$
$$y_1 \geq 0, \ y_2 \geq 0, \ y_3 \geq 0, \ y_4 \geq 0$$

4. For what values of the parameter t is $x_1 = 2$, $x_2 = 7$ an optimal solution of the following linear program?

$$\max\{t \cdot x_1 + 4x_2\}$$
subject to
$$2x_1 + x_2 \leq 16$$
$$x_1 + 3x_2 \leq 23$$
$$x_2 \leq 7$$

5. For what values of the parameter p is $x_1 = 0$, $x_2 = 4$, $x_3 = 3$, $x_4 = 0$ an optimal solution of the following linear program?

$$\max\{5x_1 + 6x_2 + x_3 + p \cdot x_4\}$$
subject to
$$2x_1 + x_2 + 2x_3 + x_4 \leq 12$$
$$x_1 + 4x_2 - x_3 + 2x_4 \leq 13$$
$$3x_1 - x_2 + 2x_3 + 3x_4 \leq 3$$
$$x_1 + 2x_2 + 4x_3 - 3x_4 \leq 21$$
$$2x_1 + x_2 + x_3 + 5x_4 \leq 7$$