Complementary Slackness – Form 2

Combinatorial Optimization – Group K

Class 18

Spring 2023

1. Decide if $x_1 = 0$, $x_2 = 26$, $x_3 = 0$, $x_4 = 1$ is an optimal solution of the following linear programs.

(a)

$$\begin{aligned}
&\max\{7x_1 + 4x_2 + 10x_3 + x_4\} \\
&\text{subject to} \\
&3x_1 + x_2 + x_3 + 4x_4 \leq 30 \\
&x_1 - 3x_2 + 2x_3 + 3x_4 \leq 13 \\
&2x_1 + x_2 + 3x_3 - x_4 \leq 25 \\
&x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0
\end{aligned}$$

(b)

$$\begin{aligned}
&\max\{7x_1 + 4x_2 + 11x_3 + x_4\} \\
&\text{subject to} \\
&3x_1 + x_2 + x_3 + 4x_4 \leq 30 \\
&x_1 - 3x_2 + 2x_3 + 3x_4 \leq 13 \\
&2x_1 + x_2 + 3x_3 - x_4 \leq 25 \\
&x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0
\end{aligned}$$

2. Decide if $x_1 = 7$, $x_2 = 0$, $x_3 = 0$, $x_4 = 2$ is an optimal solution of the following linear program.

$$\begin{aligned}
&\max\{5x_1 + 2x_2 + 3x_3 + x_4\} \\
&\text{subject to} \\
&5x_1 + x_2 + 4x_3 + x_4 \leq 37 \\
&3x_1 + 8x_2 + x_3 + 5x_4 \leq 33 \\
&2x_1 + 6x_2 + 3x_3 + x_4 \leq 17 \\
&x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0
\end{aligned}$$

3. A mobile-home manufacturer in Indiana channels its mobile-home units through distribution centers located in Elkhart, Ind., Albany, N.Y. and Camden, N.J. An examination of their shipping department records indicates that, in the upcoming quarter, the distribution centers will have in inventory 30, 75, and 60 mobile homes, respectively. Quarterly orders submitted by dealerships serviced by the distribution centers require the following numbers of mobile home units for the upcoming quarter: 25 for Dealer A, 40 for Dealer B, 15 for Dealer C, 25 for Dealer D, and 50 for Dealer E. Transportation costs (in dollars per unit) between each distribution center and the dealerships are as shown in the left-hand side table below.

Transportation costs	Dealer				
Distribution center	A	B	C	D	E
Elkhart	75	65	175	90	110
Albany	90	30	45	50	105
Camden	40	55	35	80	70

Amounts proposed	Dealer				
Distribution center	A	B	C	D	E
Elkhart	20	0	0	0	0
Albany	0	40	10	25	0
Camden	5	0	5	0	50

The manufacturer wants to minimize the total transportation cost from the distribution centers to the dealerships. Its planning department proposes to transport the amounts shown in the right-hand side table from the distribution centers to the dealerships. Should the management accept this proposal?

4. The following linear program also got severely injured in an unfortunate accident. Determine the maximum value of the program and restore the lost values (represented by $\square$'s) if we know that $x_1 = 1$, $x_2 = 2$, $x_3 = 0$ is an optimal solution of the program.

$$
\begin{aligned}
&\max\{5x_1 + \square\, x_2 + 8x_3\} \\
&\text{subject to} \\
&x_1 + x_2 + 9x_3 \leq 5 \\
&3x_1 + 4x_2 + 13x_3 \leq 12 \\
&5x_1 + x_2 + 17x_3 \leq \square \\
&x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0
\end{aligned}
$$