

An application in game theory – Part I
COMBINATORIAL OPTIMIZATION – GROUP K
Class 19
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1. Two players, Rowan and Colin play Rock-Paper-Scissors for money. The rules are: if a player wins with throwing Rock (against Scissors) then the loser pays 30 Ft to the winner; if a player wins with Paper (against a Rock), then the loser pays 20 Ft to the winner; if a player wins with Scissors (against a Paper), then the loser pays 10 Ft to the winner; finally, if they throw the same then it is a draw, there is no payoff.

Prepare yourself for playing this game against your partner.

2. The following game is a simplified model of the endgame in poker. Two players, Blue and Red put an ante of 10 Ft and 20 Ft, respectively into a pot. Blue then rolls a dice, but he keeps the result hidden from Red. Then Blue may check by saying „I threw small”, or he may bet by saying „I threw big”. In this game, „big” means 5 or 6 and „small” means anything below 5 – but of course, Blue’s statement need not be true. If Blue says „I threw small”, the game is over and the antes go to Red. If, however, he says „I threw big”, he has to put another 30 Ft into the pot. Now it is Red’s turn: she may fold by saying „I believe you” or she may call by saying „You liar!” In the first case, the game is over and Blue wins the pot (no matter what he rolled). In the second case, Red also puts another 30 Ft in the pot and the moment of truth comes: Blue shows Red what he rolled and if it was indeed big (5 or 6) then he wins the pot, if it was small then the pot goes to Red.

Prepare yourself for playing this game against your partner.

3. Redland and Blueland are in war against each other. Battles take place on two battlefields: the red battlefield and the blue battlefield (both in the respective countries). The Red King has 3 regiments, while the Blue King has 4 regiments. Both kings decide how many regiments to send to the two battlefields, then the two battles are fought separately. If the number of red and blue regiments sent to a battlefield happen to be equal then no-one wins there, both armies withdraw. If, on the other hand, one of the kings sends more regiments to a battlefield than the other, then he is the winner there: he wins 20 Ft for winning the battle and furthermore he captures all the regiments of the other king sent to that battlefield, each of which are worth 10 Ft.

For example: the Red King commands all of his 3 regiments to the red battlefield and sends no regiment at all to the blue one, while the Blue King divides his forces and sends 2 regiments to both battlefields. Then the Red King won on the red battlefield (3 red regiments against 2 blue ones), he wins 40 Ft (20 Ft for winning the battle and 20 Ft for the two regiments captured). On the blue battlefield the winner is the Blue King (two regiments against none), he wins 20 Ft (for the battle). All in all, Blue pays Red 20 Ft.

Prepare yourself for playing this game against your partner.

4. Two players, Eve and Odell play the following game. They both palm \$1 or \$2 and then they simultaneously show the hidden amounts. If the sum of the two amounts is even then the winner is Eve, if it is odd then the winner is Odell. The loser always pays the sum of the two hidden amounts to the winner (who can also keep his or her own hidden money). What are the optimal mixed strategies of the two players?

5. Rowan and Colin modify the Rock-Paper-Scissors game of Problem 1. in the following way. It still holds that the winner wins 30 Ft or 20 Ft or 10 Ft if he wins with Rock or Paper or Scissors, respectively, however, the payoff for draws is modified. Before the game starts, both players choose a „favorite” hand-sign out of Rock, Paper or Scissors; they are not allowed to choose the same. Then if the two players throw the same then the one who chose that hand-sign to be his favorite wins 10 Ft from the other; if none of the two players chose that specific hand-sign as his favorite then there is no payoff. Rowan kindly offers Colin to choose his favorite hand-sign first. Should Colin accept this offer?