An application in game theory – Part III

Combinatorial Optimization – Group K

Class 21

Spring 2023

1. Decide (without using a computer) if $x = \left(\frac{1}{4}, 0, 0, \frac{3}{4}\right)^\top$ is an optimum mixed strategy for the Column Player in the two-player, zero-sum game given by the payoff matrix below.

$$\begin{pmatrix} 8 & 6 & 9 & 4 \\ -1 & 2 & -3 & 7 \\ 6 & 20 & 21 & 5 \end{pmatrix}$$

2. The payoff matrix A of a two-player, zero-sum game is a „magic square"; that is, an $n \times n$ matrix with the property that the sum of the elements in every row or column is the same number, say, k. Show that the value of the game is $\frac{k}{n}$.

3. Three decks of cards – named A, B, and C – lie on the table face down, each contain 6 black cards (and no other cards). Red closes her eyes and Blue replaces a few black cards with white ones. Blue has three choices: either he replaces 3 cards in deck A or 2 cards in deck B or 1 card in deck C. After he is finished, he shuffles his chosen deck and puts it back to its original place (and hides the black cards he removed). Then Red opens her eyes and chooses 2 out of the 3×6 cards. If at least one of the chosen cards is white, then Red is the winner; if not, then Blue wins. The value of the winnings is the same in both cases, say, 100 Ft. What are the optimal mixed strategies of the two players? What is the value of the game?