Network flows revisited

Combinatorial Optimization – Group K

Class 22

Spring 2023

1. The incidence matrix B of a loopless directed graph G was injured in an unfortunate accident. Restore the lost values (represented by $\square$'s) and draw the graph.

$$B = \begin{pmatrix} 1 & \square & 1 & 0 \\ \square & 0 & \square & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & \square & \square \end{pmatrix}$$

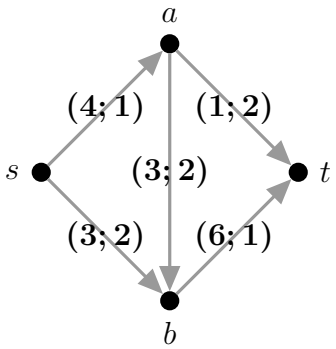
2. *(Minimum Cost Flow Problem.)* Relying on road networks again for a motivation, it is obvious that carrying a unit of flow along an arc e has a fixed cost $k(e)$. (Think of fuel costs, highway toll, wage of the driver, etc.) Assume the task is to transport a given amount of a single commodity from a given s to a given t, then it is just sensible to look for a flow for which the total cost (payed on all arcs) is minimum while given arc capacities $c(e)$ are still respected.

   (a) Based on the above rough description, give a precise definition of the minimum cost flow problem (in terms of input, output and objective).

   (b) Formulate the minimum cost flow problem as a linear program.

3. Give a linear programming formulation of the minimum cost flow problem shown in the figure. (Describe this formulation in as detailed a form as possible; that is, introduce all necessary variables and spell out all the constraints and the objective function using them.) The first numbers assigned to the arcs are the capacities, the second ones are the cost of carrying a unit of flow along the arc, and the task is to transport 6 amount of commodity from s to t.

   Solve the minimum cost flow problem by running Excel's LP solver on the obtained formulation.



4. *(Multicommodity Flow Problem.)* If we think of a directed graph as a road network, it is a natural assumption that the same infrastructure is used for transporting various commodities. Assume that k different types of commodities are given, each has its own respective source node and target node. However, all commodities contribute jointly to the load of each arc. The task is to transport some amount of each commodity between the corresponding source and target such that the total amount of commodities carried is maximized.

(a) Based on the above rough description, give a precise definition of the multicommodity flow problem (in terms of input, output and objective).

(b) Formulate the multicommodity flow problem as a linear program.

5. Give a linear programming formulation of the 2-commodity flow problem shown in the figure. (Describe this formulation in as detailed a form as possible; that is, introduce all necessary variables and spell out all the constraints and the objective function using them.)

   (a) Solve the 2-commodity flow problem by running Excel's LP solver on the obtained formulation.

   (b) Solve the integer version of the same 2-commodity flow problem (that is, each flow value has to be integer).