

The Hungarian Method – Egerváry's algorithm

COMBINATORIAL OPTIMIZATION – GROUP K

Class 24

Spring 2023

All matrices on this sheet represent weighted bipartite graphs in the following sense. The bipartite graph is $G = (A, B; E)$, where $A = \{a_1, a_2, \dots, a_m\}$, $B = \{b_1, b_2, \dots, b_n\}$ and the edge set and the edge weights are contained in the $m \times n$ matrix such that for all $1 \leq i \leq m$ and $1 \leq j \leq n$ the intersection of the i -th row and the j -th column corresponds to the edge $\{a_i, b_j\}$: X means that the edge is not present in the graph and a number means that it is present with the number being its weight.

1. Use Egerváry's algorithm to find a maximum weight perfect matching in the following bipartite graph.

$$\begin{pmatrix} 8 & 3 & 5 & 4 \\ 7 & 1 & 6 & 2 \\ 9 & 3 & 4 & 1 \\ 4 & 2 & 7 & 5 \end{pmatrix}$$

2. Use Egerváry's algorithm to find a matching of maximum weight in the following bipartite graph.

$$\begin{pmatrix} 3 & 3 & X & 7 & 5 \\ 5 & 3 & 5 & 7 & 6 \\ 1 & X & 3 & 4 & X \\ 5 & 5 & 6 & 9 & 9 \end{pmatrix}$$

3. (a) Use Egerváry's algorithm to find a maximum weight perfect matching in the following bipartite graph.
(b) Find a maximum weight matching in the same graph.

$$\begin{pmatrix} -2 & 5 & 0 & 1 \\ -2 & X & -1 & 0 \\ 1 & 6 & 2 & 4 \\ -3 & 4 & 3 & X \end{pmatrix}$$