Final Exam Review Problems

Combinatorial Optimization – Group K

Spring 2023

1. (a) Find the dual of the following linear program.

   (b) Determine the minimum value of the (primal) program. (You can rely on the fact that the system of the primal program is solvable and that its objective function is bounded from below, you don't need to prove these.)

$$\begin{aligned}
&\min\{4x_1 + 10x_2 + 6x_3 + 5x_4\}\\
&\text{subject to}\\
&4x_1 - 5x_2 + 3x_3 + 5x_4 \le 5\\
&2x_1 - x_2 - x_3 + x_4 \le 1\\
&3x_1 + 2x_2 + 2x_3 + 3x_4 \ge -2
\end{aligned}$$

2. (a) Find the dual of the following linear program and reduce it to as simple a form as possible.

   (b) Decide if the objective function of the (primal) program is bounded from above.

$$\begin{aligned}
&\max\{x_1 + x_2 + 3x_3 - 2x_4\}\\
&\text{subject to}\\
&x_1 + 3x_2 + 2x_3 - 2x_4 = 5\\
&2x_1 - 4x_2 - x_3 + x_4 \ge 3\\
&x_1 + 5x_2 - x_4 \le 6\\
&2x_1 + x_2 + x_3 - x_4 \le 10\\
&x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0
\end{aligned}$$

3. (a) Find the primal linear program he dual of which is the following linear program.

   (b) Is it true that the objective function of the primal program is bounded from above on its set of solutions? If yes, determine the maximum value of the primal.

$$\begin{aligned}
&\min\{y_1 + y_2\}\\
&\text{subject to}\\
&y_1 + 3y_2 = 5\\
&2y_1 - 5y_2 = -1\\
&3y_1 - y_2 = 5\\
&y_1 \ge 0,\ y_2 \ge 0
\end{aligned}$$

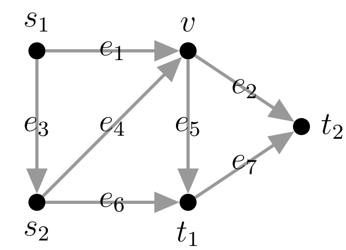
4. Decide if $x_1 = 3, x_2 = 0, x_3 = 1, x_4 = 0$ is an optimal solution of the following linear program.

$$\begin{aligned}
&\max\{21x_1 + 7x_2 + 20x_3 + 12x_4\}\\
&\text{subject to}\\
&5x_1 + 2x_2 + 4x_3 + 2x_4 \le 19\\
&3x_1 - x_2 + 6x_3 \le 17\\
&x_1 + +4x_3 + 3x_4 \le 7\\
&x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0
\end{aligned}$$

5. The payoff matrix of a two-player, zero-sum game is the $k \times n$ matrix A. There exists a pair of indices i, j ($1 \le i \le k$, $1 \le j \le n$) such that every element of the i-th row of A is less than or equal to every element of the j-th column of A. Show that the value of the game is $a_{i,j}$ (the element in the intersection of the i-th row and the j-th column).

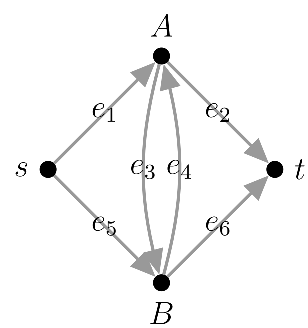
6. (a) Give a linear programming formulation of the following 2-commodity flow problem. The sum of the overall flow values is to be maximized where the source and target nodes corresponding to the two commodities are s_1, s_2 and t_1, t_2, respectively. The capacity of the arcs e_1, e_5 and e_6 is 2, while the capacity of the rest of the arcs is 1.

   (b) Solve the problem using Excel.



7. (a) Give a linear programming formulation of the following minimum cost flow problem. The capacity of each arc is 2 and the cost values corresponding to the arcs are to be seen in the table below. A minimum cost flow of overall value at least 3 is sought.

   (b) Solve the problem using Excel.



e	:	e_1	e_2	e_3	e_4	e_5	e_6
$k(e)$	:	3	3	1	1	5	1

8. *(Problem 24/3.)*

   (a) Use Egerváry's algorithm to find a maximum weight perfect matching in the following bipartite graph.

   (b) Find a maximum weight matching in the same graph.

$$\begin{pmatrix} -2 & 5 & 0 & 1 \\ -2 & X & -1 & 0 \\ 1 & 6 & 2 & 4 \\ -3 & 4 & 3 & X \end{pmatrix}$$