MINICOURSES

V. BALADI (CNRS)

Some "new" Banach spaces, and why they are useful for dynamics

The (Ruelle) transfer operator is a powerful tool to study ergodic properties of dynamical systems enjoying some hyperbolicity. In order to obtain good spectral properties, it is essential to let this operator act on a well chosen Banach space. Due to the limited smoothness of dynamical foliations, this is a difficult problem, even for Anosov diffeomorphisms. The case of analytic uniformly hyperbolic diffeomorphisms and flows was treated by Rugh and Fried in the early nineties. Handling systems with finite differentiability was first done in a pioneering paper of Blank-Keller-Liverani in 2002.
We shall present some Banach spaces introduced in the past five years or so (by Gouëzel, Liverani, Tsujii, Demers, and the lecturer), and some of their applications. We shall see that the smooth uniformly hyperbolic case is now fairly well understood (both for diffeomorphisms and flows). Piecewise hyperbolic systems pose new challenges, and for the moment we are quite far from addressing interesting billiards.
We shall restrict to systems in finite dimension.

P. BÁLINT - I. P. TÓTH (Technical University of Budapest)

An application of Young's tower method: exponential decay of correlations for multi-dimensional dispersing billiards.

Brief plan:

1.)
- Introduction to the problem of correlation decay.
- On the tower construction in general.
- Formulation and role of the growth lemma for unstable manifolds.
- Chernov's '99 theorem

2.)
- Formulation of the theorem about EDC of multi-dimensional dispersing billiards.
- Sketch of the proof.
- Choice of a suitable Riemannian metric.

3.)
- Proof and discussion of the growth lemma.
- Remarks concerning the complexity assumption about the singularity set.

N. I. CHERNOV (University of Alabama, Birmingham)

1.) Coupling methods for hyperbolic maps and flows, with and without singularities. (Markov approximations, Doeblin-type conditions, Coupling of standard pairs)
2.) One-step expansion assumptions sufficient for exponential mixing rates. (Several versions, strong and weak, that are designed for various billiards)
3.) Billiard tables with slow mixing rates that require different approaches. (Stadia, drivebelts, flower regions, boxes with obstacles, cusps, etc.)

D. DOLGOPYAT (University of Maryland, College Park)

Martingale approach to Limit Theorems for Hyperbolic Dynamical systems

We describe martingale approach to proving diffusion type limit theorems for hyperbolic dynamical systems including dispersing billiards. We explain the relation of this topic to smoothness properties of SRB measures and to recurrence time statistics in infinite measure systems.

C. LIVERANI (Università di Roma Tor Vergata)

Statistical properties of infinite dimensional systems.

I will discuss the transfer operator approach to the study of the statistical properties of infinite systems and discuss its current achievements and limits. I will discuss the problem of obtaining a rigorous derivation of the heat equation form a microscopic model and outline some strategies toward this end. Finally, I will show how the above mentioned transfer operator methods can be used to implement at least a part of such strategies.

A. WILKINSON (Northwestern University, Evanston)

From uniform to partial hyperbolicity

In this series of lectures, I will discuss classical aspects of uniformly hyperbolic theory that carry over into the partially hyperbolic setting (with a focus on analytic and ergodic properties). The goal is to present a general theory, and the common theme will be accessibility. I will focus on the following topics:

- Ergodicity and mixing properties of conservative systems
- Livsic theory and solutions to the cohomological equation
- Lyapunov exponents