Semester on Hyperbolic Dynamical Systems: Week 3 seminar abstracts

The seminars will take place on Tuesday and Thursday at the Erwin Schrödinger Institute

ABSTRACTS OF TALKS:

Felipe Barra (Universidad de Chile):

Dynamics in the Self-Similar Lorentz Channel.

(Joint work with Nikolai Chernov, Thomas Gilbert and Sebastian Reyes)

The self-similar Lorentz billiard channel is a spatially extended deterministic dynamical system which consists of an infinite one-dimensional sequence of cells whose sizes increase monotonously according to their indices. This special geometry induces a drift of particles flowing from the small to the large scales. In the near equilibrium limit the average drift velocity is determined by the diffusion coefficient like in a conductivity formula and far from equilibrium this velocity presents log-periodic oscillations.

Nicolai Haydn (University of Southern California):

The distribution of measures of cylinder sets for uniformly strong mixing measures

The theorem of Shannon-McMillan-Breiman states that for ergodic measures the measures of cylinder sets decay exponentially at a rate given by the metric entropy. We show that for countably infinite partitions and uniformly strong mixing measures the measures of cylinder sets are lognormally distributed and moreover provide (polynomial) error terms. As a corollary we also obtain the law of the iterated logarithm and we are also to prove the weak invariance principle of the information function.

Huyi Hu (Michigan State University):

Convergence rates of the transfer operators for sigma finite measures

(Joint work with Nicolai Haydn)

Consider a piecewise smooth expanding map f from the unit interval to itself such that near the origin, $f(x) = x(1 + |x|^t)$+higher order terms, where t is larger then or equal to 1. So the systems admit σ-finite an absolutely continuous invariant measure. Under the normalized transfer operator L , a Holder continuous test function g converges to $g(0)$. We will give the order of the rates of convergence.

Yuri Kifer (Hebrew University, Jerusalem):
Nonconvergence examples in averaging.

(Joint work with Victor Bakhtin)

Systems which combine fast and slow motions lead to complicated two scale equations and the averaging principle suggests to approximate the slow motion by averaging in fast variables. When the fast motion does not depend on the slow one this approximation usually works for all or almost all initial conditions but when the slow and fast motions depend on each other (fully coupled), as is usually the case, the averaging prescription cannot always be applied, and when it is valid then only in the sense of convergence in measure (or in average) with respect to initial conditions. A nonconvergence example for fixed initial conditions constructed for small perturbations of integrable Hamiltonian fast motions is due to Neishtadt and it is based on the well known phenomenon of resonances there. We construct nonconvergence examples in the discrete time averaging setup in a completely different situation where fast motions are expanding maps and Markov chains. In this case large deviations results provide an exponentially fast convergence in measure on initial conditions while for almost all fixed initial conditions there is no convergence at all. The proof for Neishtad's example requires only elementary ordinary differential equations tools but even for simplest expanding maps of the circle the proof of non convergence is not trivial and it relies on thermodynamic formalism and large deviations results. It seems that this situation is typical for chaotic fast motions but how to extend the proof to even a bit more general situation is not clear yet.

Stefano Luzzato (Imperial College, London):
Invariant measures for interval maps with critical points and singularities

I will talk about recent work with Araujo and Viana on the existence of absolutely continuous invariant measures for interval maps with non degenerate critical points and singularities under some very weak hyperbolicity conditions. The proof uses a very soft form of inducing as well as variation estimates.

Yakov Pesin (Pennsylvania State University):
Thermodynamics of towers and the liftability problem

Thermodynamical formalism of statistical physics is a collection of results aimed at producing some "natural" invariant measures with strong ergodic properties including

Sinai-Ruelle-Bowen measures (absolutely continuous invarinant measures in the one-dimensional case), measures of maximal dimension and entropy. In the classical situations (Anosov maps, hyperbolic attractors, etc.) thermodynamical formalism can be effected using symbolic representations of these systems via subshifts of finite type. Many non-classical situations (e.g., one-dimensional unimodal maps and Henon attractors) can be handled using symbolic representations via towers whose base is the Bernoulli shift on a countable set of states. I will describe general tower constructions and present some recent results on thermodynamics of the corresponding systems and in particular, on the study of the pressure function. The principle new phenomenon is that one may have to reduce the class of invariant measures under consideration to the so-called liftable measures. I will describe these measures and discuss the associated liftability problem.