Semester on Hyperbolic Dynamical Systems: Week 4 seminar abstracts

The seminars will take place on Tuesday and Thursday at the Erwin Schrödinger Institute

ABSTRACTS OF TALKS:

Thomas Gilbert (Université Libre de Bruxelles):
Equilibrium and non-equilibrium Galton boards

Galton boards are models of deterministic diffusion in a uniform external field, akin to driven periodic Lorentz gases, which are here considered in the absence of dissipation mechanism. By considering a cylindrical geometry with axis along the direction of the external field, the two-dimensional board becomes a model for one-dimensional mass transport along the direction of the external field. Equilibrium and non-equilibrium stationary states arise, depending on the specific choice of boundary conditions at the ends of the cylinder. While the former is associated to a closed board and has a uniform invariant measure, the latter is associated to an open board with the two ends in contact with particle reservoirs and has a fractal invariant measure. Numerical results are presented in support of this claim. A correspondence is established between the local phase-space statistics and their macroscopic counter-part. Analytical results are obtained for the statistics of multi-baker maps associated to such a non-uniform diffusion process and the fractality of the invariant state related to the positivity of the entropy production rate.

Vadim Kaloshin (University of Maryland):
Hausdorff dimension of oscillatory motions for the 3 body problem

Oscillatory motions in the 3 body problem are studied, involving construction of interesting hyperbolic invariant sets for area-preserving diffeos.

Marco Lenci (Università di Bologna):
On infinite-volume mixing

I will discuss a few ideas about the long-standing and deep problem of finding a definition of mixing for dynamical systems preserving an infinite measure.

Nándor Simányi (University of Alabama at Birmingham):
Homotopical Rotation Numbers of 2D Billiards

Traditionally, rotation numbers for toroidal billiard flows are defined as the limiting vectors of average displacements per time on trajectory segments. Naturally, these creatures are living in the (commutative) vector space R^n, if the toroidal billiard is given on the flat n-torus.
The billard trajectories, being curves, oftentimes getting very close to closed loops, quite naturally define elements of the fundamental group of the billiard table. The simplest non-trivial fundamental group obtained this way belongs to the classical Sinai billiard, i.e., the billiard flow on the 2-torus with a single, convex obstacle removed. This fundamental group is known to be the group F_2 freely generated by two elements, which is a heavily noncommutative, hyperbolic group in Gromov's sense. We define the homotopical rotation number and the homotopical rotation set for this model, and provide lower and upper estimates for the latter one, along with checking the validity of classicaly expected properties, like the density (in the homotopical rotation set) of the homotopical rotation numbers of periodic orbits.
The natural habitat for these objects is the infinite cone erected upon the Cantor set $Ends(F_2)$ of all "ends" of the hyperbolic group F_2. An element of $Ends(F_2)$ describes the direction in (the Cayley graph of) the group F_2 in which the considered trajectory escapes to infinity, whereas the height function t ($t > 0$) of the cone gives us the average speed at which this escape takes place.

Jacopo de Simoi (University of Maryland):
Stability and Instability results in a model of Fermi Acceleration

A bouncing ball system is an Hamiltonian system that can be used to model the mechanism underlying Fermi acceleration. We consider a ball bouncing elastically on a infinite plate that performs a sinusoidal motion. The ball is subject to a potential force that brings it back to the plate. One of the main questions about this kind of systems regards the abundance of escaping orbits, i.e. orbits such that energy grows to infinity along with time. In the talk we show that, under appropriate conditions on the potential, one has abundance of stable and unstable motions for all energies. Namely we show that, for all sinusoidal motions of the plate, the set of escaping orbits has full Hausdorff dimension. On the other hand for almost all sinusoidal motions we show how to construct stable two-periodic elliptic islands for arbitrarily high energies. In the proof of the second result we prove a Diophantine-like approximation condition that is of independent interest.

Paul Wright (University of Maryland):

Some rigorous results for the periodic oscillation of an adiabatic piston

A simple model of an adiabatic piston consists of a heavy piston of mass M that separates finitely many ideal, unit mass gas particles moving inside two gas containers. Averaging techniques, used to study the motion of the slow-moving piston in the limit where M tends to infinity, suggest that the piston should oscillate periodically. For one-dimensional chambers, the effects of the gas particles are quasi-periodic and can be essentially decoupled, and I will show that we recover a strong law of large numbers that is characteristic of classical averaging over just one fast variable: the deviation of the piston from its averaged behavior is no more than $O(M^{-1/2})$ on a time scale $O(M^{1/2})$. I will also show that for a very general gas chamber in higher dimensions, the actual motions of the piston converge in probability to the averaged behavior on that time scale, although a strong law is no longer possible. I learned about this problem from the papers of Neishtadt and Sinai, who derived the averaged equations and pointed out that an averaging theorem due to Anosov could be extended to this case.