Semester on Hyperbolic Dynamical Systems: Week 5 workshop abstracts

ABSTRACTS OF TALKS:

Jose Alves (Universidade do Porto):
Liftable absolutely continuous invariant measures

We consider smooth maps in Riemannian manifolds with finite dimension. We give a characterization of the ergodic absolutely continuous measures (with respect to Lebesgue measure) which are obtained as a projection of the absolutely continuous invariant measure of some induced Markov map.

Pavel Bachurin (University of Toronto):
TBA

Henk van Beijeren (Utrecht University):
Green-Kubo formalism for solids

The Green-Kubo formalism yields macroscopic transport equations on the basis of the microscopic equations of motion, by postulating the existence of a set of slowly varying microscopic phase functions, whose dynamics allow for a closed macroscopic description. These functions always include the long-wavelength Fourier components of energy, momentum and mass densities. In systems with broken symmetries the order parameters describing these have to be added. In solids these primarily are displacement fields, describing the displacements of the atoms from their equilibrium positions. The resulting macroscopic equations are the elastic equations describing propagation and damping of sound and Fourier's law of heat conduction. The Green-Kubo formalism expresses the transport coefficients and damping constants occurring in these equations in terms of time integrals of current-current time correlation functions. I will present the general structure of these equations together with the Green-Kubo expressions for transport and damping coefficients.

Leonid Bunimovich (Georgia Institute of Technology):
Where to place a hole to achieve a maximal escape rate

A natural question how the survival probability depends upon a position of the hole was seemingly never addressed in the theory of open dynamical systems. This dependence can be very essential and the properties of dynamics may play even a more important role than the size (measure) of the hole. The main results deal with Markov holes (i.e. with the elements of Markov partitions) and with sequences of shrinking holes for strongly chaotic maps without distortion (like tent map, baker's map, etc) and their metrically conjugate dynamical systems. The technical tools used are borrowed from symbolic dynamics and from combinatorics on words.

Mark Demers (Fairfield University):
Billiards with Holes

Introducing a small hole into the phase space of an ergodic dynamical system causes almost every trajectory to eventually escape. Despite this, such systems can have rich dynamics. For dispersing billiards with holes, we construct physically relevant invariant and conditionally invariant measures with properties analogous to those of SRB measures for closed systems. We also prove that the conditionally invariant measures converge to the smooth invariant measure in the small hole limit. This is joint work with Lai-Sang Young and Paul Wright.

Guido Gentile (Universittà di Roma La Sapienza):
Periodic solutions for a class of nonlinear partial differential equations in higher dimension

We shall consider a class of nonlinear PDEs, including the nonlinear Schroedinger equation and the nonlinear wave equation, in higher dimension, and discuss a proof of existence of time-periodic solutions, based on renormalization group methods. With respect to previous proofs of existence of quasi-periodic solutions (Bourgain and Eliasson-Kuksin), our result covers also cases where the bifurcation equation is infinite-dimensional, such as the nonlinear Schroedinger equation with zero mass, for which solutions which at leading order are wave packets can be shown to exist.

Anton Gorodetski (University of California, Irvine):
On the size of stochastic layer of the standard map

The Chirikov-Taylor standard map of the two-torus f_k is the most famous example of a symplectic twist map, and is related to an extensive list of physical problems. A longstanding conjecture claims that f_k has a stochastic layer (a transitive set of points with non-zero Lyapunov exponents) of positive measure for non-zero values of the parameter k. At the present time it is not even known whether there exists at least one value of k such that f_k has a stochastic layer of positive measure. Using the recent results on homoclinic bifurcations of area-preserving maps, we show that stochastic layer of f_k has full Hausdorff dimension for large topologically generic values of the parameter.

Sébastien Gouëzel (Université de Rennes 1):

Necessary and sufficient conditions for limit theorems in Gibbs-Markov maps

If $Z_0, Z_1, \ldots$ are independent identically random variables, necessary and sufficient conditions are known for the existence of real sequences A_n and B_n such that $(Z_0 + \ldots Z_{n-1} - A_n)/B_n$ converges in distribution towards a nonconstant random variables. These conditions are easy to write down explicitly in terms of the distribution of Z_0. For a class of very chaotic dynamical systems – the Gibbs-Markov maps–, Aaronson and Denker have shown that the very same conditions are sufficient for the convergence in distribution of suitably renormalized Birkhoff sums. Under weak regularity assumptions, we show that these conditions are also necessary: there is no exotic limit theorem for Gibbs-Markov maps.

Gerhard Keller (Universität Erlangen):

Globally coupled piecewise expanding maps with bistable behaviour

Among the simplest globally coupled systems of chaotic maps are systems of piecewise monotone interval maps coupled by their mean field. Around 1990, Kaneko studied globally coupled logistic maps. He observed (numerically) that the fluctuations of the mean field do not necessarily scale with the square root of the system size even for small coupling strength, and he coined the term "violation of the law of large numbers" for this behaviour. A bit later, Ershov and Potapov observed essentially the same phenomenon in systems of globally coupled tent maps and explained it by semi-rigorous arguments. So far there is no rigorous explanation of this phenomenon. On the other hand, Esa Järvenpää proved in 1997 for systems of analytic expanding circle maps that this phenomenon does not occur at small coupling strength, and a bit later I showed that just C^2-smoothness of the invariant densities is needed.
In my talk I plan to first summarize these results in a probabilistic and functional analytic framework (de Finetti's theorem, self consistene Perron-Frobenius operators (SCPFO)), and then I will report on some work in progress (with J.-B. Bardet and R. Zweimüller) where we study a parametrized family of two-to-one uniformly expanding piecewise fractional linear maps of an interval. The maps are coupled via the parameter which is a sigmoid function of the mean field. Since the maps leave a space of probability measures invariant whose distribution functions are Herglotz functions, the Perron-Frobenius op-

erators of the maps have some "hidden" monotonicity properties. This allows to detail the dynamics of the (nonlinear) SCPFO for a broad range of coupling strengths and to show that the operator has a bifurcation from a unique stable fixed point to a pair of stable fixed points separated by a kind of hyperbolic fixed point. For all these parameters the finite systems have a unique mixing smooth invariant density. The dynamics of the SCPFO can be related to the large deviations behaviour of the finite systems at fixed time when the system size tends to infinity. The large deviations behaviour of the invariant densities is still unknown to us.

Rainer Klages (Queen Mary College, University of London):

Deterministic random walks in maps and billiards

I will discuss the parameter dependence of diffusion in deterministic random walks generated by two different types of hyperbolic dynamical systems. First I will outline a theory by which the deterministic diffusion coefficient in a piecewise linear map lifted periodically onto the real line can be calculated exactly. The result is found to be a fractal function that is Lipschitz continuous up to quadratic logarithmic corrections [1]. This fractality can be understood by analysing Markov partitions, the invariant probability density of the map and de Rham-type functional equations. I will then present computer simulation results and a correlated random walk approach suggesting that similar irregular diffusion coefficients exist in the periodic Lorentz gas and related Hamiltonian particle billiards [2].

[1] G.Keller, P.J.Howard, R.Klages, Nonlinearity **21** (2008), in print

[2] R.Klages, Microscopic Chaos, Fractals and Transport in Nonequilibrium Statistical Mechanics (World Scientific, Singapore, 2007)

Dong Li (Institute of Advanced Study, Princeton):

Complex blowups of some fluid dynamics equations

I will explain recent joint work with Ya.G. Sinai on constructing singularities in complex-valued solutions of the Navier-Stokes system (NSS) on R^3.

Francois Ledrappier (University of Notre Dame):

Fluctuations of the ergodic sums for the horocycle flow on abelian covers of hyperbolic surfaces

We study the almost sure asymptotic behavior of the ergodic sums of L^1–functions, for the infinite measure preserving system given by the horocycle flow on the unit tangent bundle of a $\mathbb{Z}^d$– cover of a hyperbolic surface of finite area, equipped with the volume measure. We prove rational ergodicity, identify the return sequence, and describe the fluctuations of the ergodic sums normalized by the return sequence. One application is a 'second order ergodic theorem': almost sure convergence of properly normalized ergodic sums, subject to a certain summability method (the ordinary pointwise ergodic theorem fails for infinite measure preserving systems).

Roberto Markarian (Universidad de la República, Uruguay):
Non-conservative billiards. Dominated splitting

A particle moves along straight lines inside a billiard table and when it hits one of the walls with angle A with respect to the normal line, it is reflected with angle L A (with L small than or equal to 1).
We give formulas for a general class of these *pinball billiards*. Then we restrict the analysis: the reflection angle only depends on incidence angle (not on the boundary position).
We prove that in many of billiard tables (in particular some in which classical billiard map is not hyperbolic) the dynamics has a weak form of hyperbolicity called dominated splitting (the tangent bundle splits into two invariant directions, the contractive behavior on one of them dominates the other one by a uniform factor).
Joint work with Enrique Pujals and Martín Sambarino.

Jens Marklof (University of Bristol):
The Boltzmann-Grad limit of the periodic Lorentz gas

The periodic Lorentz gas describes a particle moving in a periodic array of spherical scatterers, and is one of the fundamental mathematical models for chaotic diffusion in the periodic setting. The plan for these lectures is to describe the nature of the diffusion when the scatterers are very small, and prove the existence of a stochastic process that governs the limiting dynamics. The problem is closely related to some basic questions in number theory, in particular the distribution of lattice points visible from a given position, cf. Polya's 1918 paper "on the visibility in a forest" (Polya's orchard problem). The key technology in our approach is measure rigidity, a branch of ergodic theory that has proved valuable in recent solutions of other problems in number theory and mathematical physics, such as the value distribution of quadratic forms at integers, quantum unique ergodicity and questions of diophantine approximation. The results presented are based on joint work with A. Strombergsson, Uppsala.

Ian Melbourne (University of Surrey):
Large and moderate deviations for slowly mixing nonuniformly hyperbolic systems

In previous work with Matthew Nicol (Houston) we obtained essentially optimal large deviation estimates for a large class of nonuniformly hyperbolic systems: namely those modelled by Young towers with summable decay of correlations (both exponential and polynomial).
In this talk I will describe recent improvements. In particular, large (and moderate) deviation estimates are obtained for arbitrarily slow polynomial decay rates. A byproduct of the proof is improved large deviation estimates even in the case of summable decay. Applications include many examples of planar billiards and Lorentz flows.

William Ott (Courant Institute, New York University):
Dissipative homoclinic loops and rank one chaos

We show that when subjected to time-periodic forcing, certain flows with dissipative homoclinic loops admit nonuniformly hyperbolic attractors with strong stochastic properties. The proof makes use of the recent theory of rank one maps developed by Wang and Young.

Francoise Pene (Université de Bretagne Occidentale):
Some properties of the planar Lorentz gas

The Lorentz gas is a dynamical system of point particles colliding fixed obstacles. We are interested in the planar model with periodic configurations of obstacles (and with finite horizon). We will recall the link between this model and the corresponding Sinai billiard (in the torus). We will see how the stochastic properties of the Sinai billiard are useful to study the Lorentz gas. In particular, we will talk about recurrence, ergodicity, the Lorentz gas in random scenery, the range of the Lorentz gas and the return time in small balls.

Marc Pollicott (University of Warwick):
Large deviations for intermittent maps

Large deviations measure the size of the set of points whose Birkhoff averages of functions deviate from the the integral by a fixed margin. We consider this problem for the Manneville-Pomeau map (and related examples) and Hölder continuous functions. These are classical examples of intermittent maps. This is joint work with Richard Sharp (Manchester).

Andrew Török (University of Houston):
Extreme value distributions for non-uniformly hyperbolic dynamical systems

(Joint work with Mark Holland and Matthew Nicol)

We study the Extreme Value Distribution for observations over discrete and continuous non-uniformly hyperbolic systems. We prove that for certain classes of interval maps (Young towers with exponential tails, Gibbs-Markov maps, intermittent maps that mix rapidly enough, etc.) and regular observations with finitely many maxima in a full measure set, the behavior is the same as in the i.i.d case. To obtain this, we refine the technique of Collet. The result are then extended to suspensions over these systems by a lifting theorem.

Masato Tsujii (Kyushu University, Japan):
Quasi-compactness of transfer operators for contact Anosov flows

For any C^r contact Anosov flow with $r \geq 3$, we construct a scale of Hilbert spaces, which are embedded in the space of distributions on the phase space and contain all C^r functions, such that the transfer operators for the flow extend to them boundedly and that the extensions are quasi-compact. Further we give explicit bounds on the essential spectral radii of the extensions in terms of the differentiability r and the hyperbolicity exponents of the flow.

Tamás Varjú (Technical University, Budapest):
TBA

Maciej Wojtkowski (University of Warmia and Mazury):
Abstract fluctuation theorem

We formulate an abstract fluctuation theorem which sheds light on mathematical relations between the fluctuation theorems of Evans and Searles, and Gallavotti and Cohen on the one hand, and those of Bochkov and Kuzovlev, and Jarzynski on the other.

Tatiana Yarmola (Courant Institute, New York University):
An example of a pathological random perturbation of the Cat Map

I will discuss degenerate random perturbations, where transition probabilities are smooth in some but not all directions. Under such perturbations, many dynamical systems will still have invariant densities, but some pathological things can also happen. I will discuss one such example, in which a simple perturbation of the Cat Map leads to a "global statistical attractor" in the form of a line segment, meaning all initial distributions are attracted to a measure supported on this line segment.