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Nuclei in finite non-Desarguesian projective planes. (English)
De Clerck, Frank (ed.) et al., Finite geometry and combinatorics. The 2nd international conference at Astene-Deinze, Belgium, 31 May - 6 June 1992. Proceedings. Cambridge: University Press, (ISBN 0-521-44850-6). Lond. Math. Soc. Lect. Note Ser. 191, 405-412 (1993).

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Let π be a finite projective plane of order q (not necessarily a prime power) and K a set of points of π. A point p of π is called a nucleus of K if no line through p intersects $K\backslash\{p\}$ in more than one point. A nucleus p is called an external nucleus if $p \notin K$ and an internal nucleus if $p \in K$. If K admits an external nucleus, then $|K| \leq q+1$ and if K has an internal nucleus, then $|K| \leq q+2$. Define the number n by $q = n(n+1)$. The author determines upper bounds for the size of the set $E(K)$ of external nuclei of K and for the size of the set $I(K)$ of internal nuclei of K. In detail: If $|K| = q+1$ and if the points of K are not on a line, then $|E(K)| \leq n^3 + n^2 + n + 1$. In the desarguesian case the sharper bound $|E(K)| \leq q-1$ is known. The result for the set $I(K)$ goes as follows: if $|K| = q+2$ and $K\backslash I(K)$ is a nonempty subset of the set of points of a line, then $|I(K)| \leq n^2$ if q is even and $|I(K)| \leq q-2$ if q is odd, respectively. The author describes a possible construction for a set K with $|I(K)| = q-2$ in the case q odd.
As an application of his result on $|E(K)|$ the author generalizes the following theorem to the non-desarguesian (in the sense of general) case: if $\{(P_1, L_1), \dots, (P_{q+1}, L_{q+1})\}$ is a maximal strong representative system in π (i.e. $P_i \in L_j \Leftrightarrow i = j$ and the set is maximal with respect to this property), then the lines $L_1, \dots, L_{q+1}$ are concurrent or the points $P_1, \dots, P_{q+1}$ are collinear.

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