Convex Geometry

2nd midterm - SAMPLE

1) A compact, convex set $K \subset \mathbb{R}^n$ is called *strictly convex* if its boundary does not contain a segment. Show that K is strictly convex if and only if every boundary point of K is an extremal point. (5 points)

2) Can the given sets be separated by a line? If yes, find a separating line. (5 points)

$$A = \{(-1,1),(0,-1),(4,1)\}, \qquad B = \{(-3,1),(1,-2)\}$$

3) Show that if $K_1, K_2, \ldots, K_m \subseteq \mathbb{R}^n$ are closed, convex sets and $\bigcap_{i=1}^m K_i \neq \emptyset$, then $\chi\left(\bigcup_{i=1}^m K_i\right) = 1$. (5 points)

4) Using Euler's theorem prove that the coordinates of the f-vector $f = (f_0, f_1, f_2, 1)$ of a 3-dimensional convex polytope satisfy the inequalities:

$$\frac{f_0}{2} + 2 \leq f_2 \leq 2f_0 - 4; \quad \frac{3f_0}{2} \leq f_1 \leq 3f_0 - 6. \text{ (5 points)}$$