14.04.2023

Convex Geometry

Midterm 1

1) Consider an $n \times n$ matrix M as a point of the Euclidean space $\mathbb{R}^{n^2}$ ($M = [a_{ij}]$ corresponds to the point $(a_{11}, a_{12}, \ldots, a_{nn})$). Let $\mathcal{F}$ be the family of $n \times n$ symmetric and positive semidefinite matrices. Prove that $\mathcal{F}$ is a closed, convex set. (5 points) (Recall that a matrix A is positive semidefinite if for any vector $x \in \mathbb{R}^n$, $x^T Ax \geq 0$.)

2) Prove that if m points are given in the plane such that for any three of them there is a closed unit disk $x + B^2$ containing them, then there is a closed unit disk containing all the points. (5 points)

3) Let C be a unit square, and let C' be a rotated copy of C by $\frac{\pi}{4}$. Compute the perimeter and the volume of $C + C'$. (5 points)

4) Let $S = [-p, p]$ be a closed segment in $\mathbb{R}^n$. Compute the support function of S. (5 points)