Algebra 2 **Exam items for terminal** **2020**

1. Definition of the order modulo m and its properties. Definition of the primitive root, theorem about the existence of primitive roots. Definition of the discrete logarithm and its properties. The theorem about the solution of the congruence $x^k \equiv a \bmod p$.

2. Definition of the quadratic residue and non-residue. Definition of the Legende symbol and its properties. The quadratic reciprocity law. Definition of the Jacobi symbol, and its properties. The theorem about the solvability of polynomial congruences.

3. Definition of Fermat numbers and Mersenne numbers. The prime divisors of the Fermat and Mersenne numbers. Fermat primes and Mersenne primes. Goldbach's conjecture. Dirichlet theorem about primes in arithmetic progressions. The prime number theorem. Asymptotic formula for the nth prime p_n. Chebyshev theorem.

4. Multiplicative and additive arithmetic functions. Examples: $d(n)$, $\sigma(n)$, $\omega(n)$, $\Omega(n)$ and its explicit forms. Summation formula for the Moebius function and Euler's totient function. Definition of perfect numbers. The theorem for even perfect numbers.

5. Theorem about the solutions of linear diophantine equations. Theorem for Pythagorean triplets. Gaussian integers. List of Gaussian primes. Characterization of integers that can be written as a sum of two squares.

6. Definition of field extensions. The product theorem for the degree of field extensions. Definition of finite, algebraic and separable extensions with examples. Definition of the splitting field and the normal extension with examples.

7. Definition Galois extension and Galois group with examples. Fundamental theorem of Galois theory. Application: Solvability by radicals.

8. Constructions by ruler and compass with examples: the problem of duplication of a cube, trisection of an angle, quadrature of a circle. Cyclotomic extension. Theorem about the construction of a regular n-gon.