Algebra 2 **Exam items for terminal** **2023**

1. Field extensions, degree, algebraic elements, splitting field. Algebraically closed field, algebraic closure. Existence and uniqueness.

2. Normal extension and separability. Characterisation of Galois extensions. Primitive Element Theorem.

3. Fundamental Theorem of Galois Theory. Examples. Cyclotomic polynomials. Gauss' Theorem. Finite Fields. Solvability by radicals. Chebotarev's Density Theorem.

4. Modules. Sub- and factormodules, direct sums and free modules. Submodules of free modules over PID's, their factor modules and the Smith Normal Form. Invariant factors and elementary divisors. Fundamental Theorem of Finitely Generated Modules over Principal Ideal Domains. Applications for $\mathbb{Z}$ and for $F[x]$.

5. Integral elements of rings over subrings. Ring of integral elements. Conceptual parallels with algebraic extensions. Ring of algebraic integers, Ω. Number fields, $\mathcal{O}(\mathbb{Q}(\alpha))$. Integrality and the minimal polynomial. Norm and trace.

6. Discriminant of a basis, integral basis, discriminant of a number field. Quadratic fields, their characterisation with square free integers. Integral bases and discriminants. Ideals in $\mathcal{O}(\mathbb{Q}(\alpha))$. Finite generation, ideal bases. Maximal and prime ideals. Division and containment. Unique factorisation of ideals.

7. Obtaining larger structures from smaller ones: $\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R} \to \mathbb{C}$. Cauchy sequences and null sequences. Completion with respect to $|\cdot|$. Field of p-adic numbers: completion with respect to $|\cdot|_p$.