# Convex Geometry (BMETE94AM18)

## Topics for the final exam

## Affine geometry and convex combinations

Affine and convex sets, affine dependence and independence, affine and convex combinations, affine hull and its properties, convex hull and its properties

### Theorems of Radon, Carathéodory and Helly

Theorems of Radon, Carathéodory and Helly, their applications and variants: colorful Carathéodory theorem, Jung's theorem, etc. Linear functionals, their connection with scalar product and hyperplanes.

#### Minkowski sum, separation

Minkowski addition and its properties. Support function. Isolation and separation, isolation theorem for open convex and arbitrary convex sets, separation theorems.

#### **Extremal and exposed points**

Extremal and exposed points, connection between extremal points and linear functionals, Krein-Milman Theorem. Straszewicz Theorem. Faces of convex sets.

#### Indicator function of convex bodies, Euler characteristic

Indicator function of closed, convex sets and its properties. Algebras of closed/compact convex sets, valuations, Euler characteristic and its properties. Convex polytopes, polyhedral sets and their connections. Face structure of polytopes. The *f*-vector of polytopes. Euler's theorem.

## Polarity

Polarity and its properties. Polar of a polytope. Dual polytope and its properties.

#### Moment curve, cyclic polytopes

Moment curve, cyclic polytopes. Face structure of cyclic polytopes. Gale's evenness condition. Applications.

#### Hausdorff and Banach-Mazur distance

Family of convex bodies as a metric space. Hausdorff distance of sets. Affine transformations and their properties. Banach-Mazur distance and its properties.

## Ellipsoids

Ellipsoid as an affine ball. Intersection and projection of ellipsoids. John and Löwner ellipsoids. Their application to estimate the Banach-Mazur distance of convex bodies. Isotropic position.