DIFFERENTIAL GEOMETRY FINAL EXAM

(1) Definition of topological manifolds and embedding topological manifolds into Euclidean spaces. Definition of differentiable manifolds, submanifolds, smooth maps, immersions, submersion. Singular point, regular point, critical value, regular value. The preimage of a regular value is a smooth submanifold.
(2) Sard theorem.
(3) Tangent space of a smooth manifolds, derivative of a smooth map, directional derivative, and derivations. Directional derivative in local coordinate charts.
(4) Vector fields, tensor fields and their characterizations, differential of a map, tensor product, gradient of a smooth function and its form in local coordinates. Differential forms, wedge product.
(5) Lebesgue integral on manifolds, in local coordinates and Gram determinant. Orientation of manifolds, volume form, integral of forms, line integral and flux in Euclidean space. Differentiating forms.
(6) Manifolds with boundary, induced orientation, Stokes theorem. Special cases in Euclidean spaces.
(7) Lie algebra of vector fields, integral curves of a vector field, the flow generated by teh vector fields, Lie derivative of a vector field.