Finding and Representation of Minimum Cuts and Some Applications

Abstract

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The topic of the thesis was minimum cuts in graphs – algorithms solving the minimum s-t cut problem, two representation methods of minimum cuts in undirected, weighted graphs and some applications.

Some well-known, effective algorithms exist for the minimum s-t cut problem on directed graphs, like the algorithm of Ford and Fulkerson presented in 1954 and the push-relabel algorithm of Goldberg and Tarjan presented in 1988. During the research we discovered a fairly new algorithm presented by Hochbaum in 2008. It solves the minimum cut problem first – with the equivalent maximum blocking-cut problem – and then finds the maximum flow. We made an observation about this algorithm, that it may be formulated directly on the network and it may not necessary to use a new structure.

For the general minimum-cut problem on undirected graphs we discussed two algorithms: Karger's randomized algorithm from 1993 and the contraction algorithm of Wagner and Stoer using maximum adjacency search presented in 1997. The latter one is deterministic, so it is used more frequently.

Representation of minimum cuts is also an interesting research area, because there are many remaining questions. The representation methods discussed in this work – the Gomory-Hu tree and the cactus graph – works only on undirected graphs, although the minimum s-t cut can be found in case of directed graphs too. So one interesting research area to carry out is to find a representation method for minimum cuts in directed graphs.

There are many applications of minimum cuts like graph clustering, mining, image segmentation and communication networks. Some of these fields have their own accustomed algorithms, although an other algorithm may work more efficiently. A good example for this is open-pit mining, where the algorithm of Lerchs and Grossmann is used, though researchers showed that the push-relabel method would work faster. There are some other cases, when an algorithm is improved such that it works faster in practice at the expense of worsening theoretical running time.

So this field of graph theory has undiscovered and unexploited areas, where some improvement can be performed in the future.