Abstract

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The main topic of the thesis is about extending partial automorphisms of finite graphs. The first result of this field was Hrushovski's, who proved that a graph G can be embedded into a bigger graph H such that for an arbitrary partial automorphism ψ of G there is $\sigma \in Aut(H)$ which extends ψ. This theorem is also related to the automorphisms of the Rado-graph in terms of that it is equivalent to the existence of a somewhat special type of automorphism. We introduce two special classes of automorphisms of the Rado-graph, the generic and strongly generic classes. If we define a topology on the automorphism group of the Rado-graph, we consider an automorphism generic if its conjugacy class is dense. Strongly generic automorphisms are generic automorphisms with one additional property which makes them to be conjugate, which means that if g and f are two strongly generic automorphisms of the Rado-graph R, the there is $h \in Aut(R)$ such that $f = h^{-1} \circ g \circ h$. We prove that there is a generic automorphism that is not strongly generic and also we construct a strongly generic automorphism.

Our first generalization related to the finite case is that if we embed this graph G into H such that every partial automorphism of G extends to not a proper automorphism but an endomorphism of H, can we do this in a way, that H has some nice properties, that is, every partial automorphism of H can be extended to an endomorphism of H. This problem remains unanswered and is still open for further investigations.