ABSTRACT

Spectral clustering and strategic interaction

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This thesis studies two seemingly different topics, spectral clustering and strategic interaction, but some relation between them is revealed. The idea of using spectral clustering came from the fact that both social networks and spectral clustering use graphs to represent data/agents and its connections. In strategic interaction the aim is always to reach the maximal payoff simultaneously for every agent, while minimize the costs and to find an equilibrium or multiple equilibria. In a network when we search equilibrium it can happen that not every agent is active. Spectral clustering can be used to separate active agents, similar to separate data points into an active and a non-active cluster.

After the introductory Section 1, Section 2 is about spectral clustering. Spectral clustering is an increasingly popular method for clustering data into groups, using linear algebra methods as well as fundamental statistics or graph theory, when it is necessary. Given a set of data points and some notion of similarity between all pairs of data points, the aim of clustering is to divide them into several groups such that, points in the same group are similar and points in different groups are different from each other. Without any other information we usually use similarity graphs, where data are the vertices and similarities are represented by weighted edges. I define three kind of a Laplacian matrix, and write about their characteristics and advantages. With the help of Laplacian matrices, spectral clustering algorithms are presented as well as the connections between spectral clustering, the mincut problem and random walks. I also mention the modularity matrix and the Newman-Girvan modularity.

In Section 3, I calculate the adjacency and Laplacian spectra of some well-known graphs and their complements. We need the smallest eigenvalue of the original graph and the largest eigenvalue of its complement in order to be able to determine the type of equilibria in strategic substitute games.

In Section 4, I write about strategic interactions. I define the two classes of strategic interaction games, the strategic complementary game and the strategic substitute game. In the games of strategic complements, an increase in the actions of other players leads a given player's higher actions. Games of strategic substitutes are such that the opposite is true, so an increase in his neighbors' actions forces the given player to decrease his actions. In the case of strategic complements with linear-quadratic payoff there is a condition which ensures a unique, inner equilibrium. In the case of strategic substitutes with linear-quadratic payoff there is no such condition, but we can distinguish four cases according to a utility parameter, where there is inner or corner equilibrium or sometimes there are multiple equilibria.

In Section 5, I show some applications of the algorithm introduced in Section 4, for finding all the equilibria depending on the utility parameter.