Abstract

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Bootstrap percolation is a model that belongs to the more general category of cellular automata introduced by von Neumann. It was first published by Chalupa, Leath and Reich in 1979 in the context of disordered magnetic systems, and has been studied extensively by mathematicians and physicists as well as by computer scientists and sociologists among others. It is a process taking place on a graph, where all the vertices are occupied or vacant, and a vertex becomes occupied if it has at least k occupied neighbours for some fixed k. We get a stochastic model if we take the initial set of occupied vertices at random: each vertex is occupied with some probability p, independently of each other.

The main goal of this diploma thesis is to prove results concerning the noise sensitivity or insensitivity of bootstrap percolation models. This means that we consider an initial configuration on a large graph where the above process results in complete occupation, and study if we can predict the same behaviour when we change the initial state of a small portion of the vertices.

In Section 2 we give the detailed description of the model and mention some early results. Then we study the graphs we are interested in in more detail: the integer lattice and torus in Section 3, infinite trees and in particular regular trees in Section 4, and the random regular graph in Section 5. The main feature we study is the critical window: the interval of the values of p at which the probability of complete occupation jumps from near 0 to near 1. We use these results when we study the noise sensitivity at the critical density: where the probability of complete occupation is 1/2.

After that, in Section 6, we introduce the concept of noise sensitivity and noise stability, and mention several methods that can be used to prove if they hold, demonstrating them on simple examples. In Section 7, we present our own results. We prove that bootstrap percolation on the random d-regular graph is noise insensitive in some sense when $2 \leq k \leq d-2$. Then we present our argument for noise sensitivity in the case of the integer lattice and the torus in any dimension $d \geq 2$ when $k = 2$, which uses an unproven generalization of a technique that we conjecture to work in this case.