A new solution concept for the risk allocation problem

Thesis extract

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2014

The risk allocation problem is about how to divide the effects of diversification appearing in a portfolio, among the financial instruments included in the portfolio. A solution for such a problem should be fair, while for the applicability for practical purposes a moderate computation need is also a requirement. Naturally there exists a trade-off between these two aspects.

For example, turning to solutions of risk allocation games, the Shapley value happens to perform well in the former (in terms of fairness described with meeting axioms), while most of the existing approximation methods for the Shapley value perform well in the latter requirement. Therefore the goal of this thesis is to introduce a new solution concept for the risk allocation problem, which provides good results in both mentioned fields.

This goal is achieved by introducing a new solution on the class of risk allocation games, from which a solution (risk allocation rule) for the risk allocation problem can be derived. The introduction of our new solution concept for risk allocation games is done through two steps.

First, we introduce a concept that fits well analytically, evaluating the solution on the basis of the axioms it meets, but can be used in practice only for very special problems. One of the basic ideas already appears in the formulation of this first method: there is a special structure in risk allocation problems, allowing us to approximate the Shapley solution of a risk allocation game using the Shapley solution of another risk allocation game with less players. Axiomatically we achieve a good result, from the aspect of complexity we find that the player set needs more aggregation.

The other most important idea is the introduction of weakened axioms. Generalizing the above mentioned concept, we propose a new solution with the possibility of narrowing the player set arbitrarily, leading to complexity and satisfaction of the weakened axioms depending entirely on the underlying problem and the aggregation we apply for the player set. The main result is, that we can get close to satisfying the axioms; nevertheless we might have to pay for that by computation time. Hence the trade-off is still there, but the situation appears to be better. Not only that the trade-off is somewhat quantified, but from another aspect, we can fulfill any prescribed error in the satisfaction of certain axioms.

While from the viewpoint of approximating the Shapley value, our contribution is the evaluation of approximations by the satisfaction of axioms, while keeping the computation needs reasonably low - our new solution achieves good results in these fields. It seems that we took a step from pointwise convergence in approximations of the Shapley value to convergence in norm, but regarding the evaluation of such an approximation in standard vector norms, the establishment of a bound is just partially done.