# Tied Favorite Edges for Random Walks
# Abstract

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In my thesis I observe properties of a symmetric random walk on a special class of graphs called spider graphs. Members of this class are generated by joining $m$ half-lines at their end-points, the half-lines represented by the non-negative integers as vertexes with edges between neighboring numbers.
The focus of the paper is on the edges the walk stepped over, specifically the ones crossed the most times ($\mathcal{K}_i$), called favorite edges, until any given time $i$.
In the thesis we first see that, during any realization of the random walk, there are almost surely infinitely many times when the favorite edge-set has at least two members:

$$\sum_{i=1}^{\infty} \mathbb{I}\left(|\mathcal{K}_i| \geq 2\right) = \infty$$

Whether the number of times $|\mathcal{K}_i| \geq 3$ is infinite or finite is an open problem, but it is conjectured to be the latter.
The main result of the paper is that the size of the favorite edge-set is greater than four almost surely only finitely many times:

$$\sum_{i=1}^{\infty} \mathbb{I}\left(|\mathcal{K}_i| \geq 4\right) < \infty$$

The proof is strongly reliant on the fact that if we stop the random walk when a fixed edge is crossed a fixed number of times, the numbers the edges were stepped over can be described by Galton-Watson branching processes originating from the fixed edge.