Discrete Phase-type distributions and Markov Arrival processes

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Abstract

Phase-type distributions (PH) and Markovian Arrival Processes (MAP) are widely applied in telecommunication models and queuing related models. We know a lot about their continuous time realizations CPH and CMAP, and discrete time models have poor literature compared to them. We tried to fill in this gap, and provide the discrete time counterparts of the known methods, formulas.

Discrete time models are applied in the cases when we can easily discretize time. For example if a network uses a transmission protocol with fixed packet size. In this case sending or receiving a packet can be the time unit. The Asynchronous Transfer Mode (ATM) traffic models often use DMAPs in that manner. We can also determine time deltas for continuous models and can use discrete time for modeling.

In my thesis our results of discrete models are summarized and compared to the continuous time formulas, methods. An introduction into Matrix Geometric (MG) and discrete time Phase-type (DPH) distributions is given. Their probability mass function, factorial moments are computed and a method for fitting MG to factorial moments is also provided. A connection of DPH and CPH is derived and that explains some of our difficulties. Then our main results about the canonical form of DPH(2) and DPH(3) is shown. The importance of canonical forms are also pointed out.

The other half of the thesis is about discrete time rational arrival processes (DRAP) and DMAP. The joint distribution and joint factorial moments of the arrivals are computed. A method for fitting DRAP to factorial moments is described that uses the MG fitting method, since the arrivals have MG/DPH marginals. The correlation of two arrival is also computed. Then a canonical form of DMAP(2) processes is described.

At the end we introduce the marked RAP and MAP (DMRAP and DMMAP), which are generalizations of DRAPs and DMAPs. In the appendix the continuous time formulas and methods are briefly described. These methods are implemented as part of the BuTools program package in Matlab and Mathematica languages. A copy of the current version of BuTools is attached on a disc.