The k-set problem

Abstract

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In this thesis we examine the so-called *k-set problem* which is one of the most challenging open questions in combinatorial geometry.

Consider a set P with n points in the d-dimensional space $\mathbb{R}^d$ in general position, that is, there are no $d+1$ points on a common hyperplane. Call a k-point subset $P' \subseteq P$ a *k-set of P* if there exist an open half-space H such that $P' = P \cap H$ and $|P'| = k$. Our goal is to determine the largest possible number of k-sets of a set of n points, as a function of n and k. The simplest and most studied version of the problem is set in the two-dimensional Euclidean plane, however, there is still a big gap between the best lower and upper bound even in this case, and in higher dimensions, determining the optimal asymptotic bound on the number of k-sets seems even more challenging.

In higher dimensions the k-set problem is even harder than in the plane, nonetheless there are several results. The most studied case is when $k = n/2$, that is finding the maximum number of *halving hyperplanes*.

If we interchange the role of the points and the lines with a suitable dual transformation, the k-set problem can be rephrased as the *k-level problem*, which is essentially equivalent with the k-set problem.

In Chapter 2 we will review the planar results so far, present some of the more interesting proofs. In the later chapters we mention the generalized cases and possible applications of the problem in computational geometry as well.