Abstract
Frozen percolation and random graphs

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Critical frozen percolation is percolation modified in such a way that infinitely big components are deleted from the graph at the time of their birth. In this thesis we examine the critical frozen percolation started from a configuration model with the underlying graph being the complete graph. Our main goal was to find the local limit of the graph sequence whose n^{th} element is the graph $G^{(n)}(t)$, the graph in the frozen percolation started from a configuration model on n vertices at time t.
We have thus investigated $r_T(t)$, the limit of $r_T^{(n)}(t)$ which is the ratio of the number of the copies of the graph T in the n^{th} element of the graph sequence and n. We proved that if the random variables $r_T^{(n)}(t)$ converge in probability for every graph T to deterministic numbers, then the Benjamini-Schramm limit exists if the size of the connected component of a uniformly chosen vertex in graph $G^{(n)}(t)$ is tight.
We call hybrid graphs the family of graphs whose edgeset is the union of the edgeset of an Erdős-Rényi graph and a configuration model on the same vertex set. We have proved the existence of r_T and calculated r_T for every T in the hybrid graph model and also in the critical frozen percolation model started from a configuration model. We have shown that they are the same if we choose the parameters of the hybrid graph in a given way. This allowed us to reach our main goal and prove that the Benjamini-Schramm limit of $G^{(n)}(t)$ is a two type branching process. We have also shown that if we start critical frozen percolation from a configuration model, then $r_{(\cdot)}^{(n)}(\cdot)$ converges in probability uniformly in time in the l^1 topology to the unique solution of the natural generalisation of Stockmeyer's differential equation. Our method was the same as the method of the authors of [1].

The limit of the ratio of the number of the remaining vertices in $G^{(n)}(t)$ and n is called the total mass at time t. We proved the existence of this limit in probability for every t. Under the assumption that the total mass is differentiable and strictly decreasing in time, we proved that the frozen percolation started from a critical configuration model remains critical.

References

[1] R. Normand and M. Merle, *Self-organized criticality in a discrete model for Smoluchowskis equation*, http://arxiv.org/pdf/1410.8338.pdf, 2015.