Random walks and the Frog Model

Abstract

Alexander Biró

Supervisor: Dr. Gergely Kiss

We start by recalling basic concepts of probability theory and stochastic processes, and also proving two simple theorems that will reflect the motives of a proof found in the later sections.

This is followed by outlining the well-known facts about simple random walks, and we discuss the questions arising about recurrence of the aforementioned process when observed on the standard lattice in d–dimensions by following one of the classic proofs of Pólya's theorem using generating functions as an extremely handy tool.

The third chapter deals with a relatively new model where one moving particle sets the others into motion. Here, recalling the statement about the recurrence in the symmetric case is followed by the recent proof of dichotomy when the particles have a drift. Finally, in a random initial setting, even when they tend away from the origin, the condition $\mathbb{E}\{\log^+ X\} = \infty$ guarantees the returns.

The second part of the chapter deals with the „modified frog model", when the density of the starting configuration of sleeping particles follow Bernoulli(p). We identify a critical rate of decay of $p(x)$ separating recurrence from transience, and study some other properties of the model.

In the last chapter we list a few applications of random walks and the frog model.