# ABSTRACT

## The Hausdorff measure of self-similar fractals of the plane



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The Hausdorff measure is one of the most basic concepts in fractal geometry. Yet, estimating its exact value is extremely difficult, even for the most simple fractals. This thesis aims for collecting the most relevant results for the topic, with exceptional attention to the Hausdorff measure of the Sierpinski triangle.

First we provide the definitions needed to describe this topic, including but not limited to: Hausdorff measure, Hausdorff dimension, self similarity, open set condition. We also define the attractor of an iterated function system, while showing some examples.

Then we summarize some of the theoretical results of the topic, namely we prove that the Hausdorff measure of the middle third Cantor set satisfies $\mathcal{H}^s(\mathcal{C}) = 1$, where $s = \log 2/\log 3$, and we prove that if the similarity ratios satisfy $\lambda \leq 1/4$, then the Hausdorff measure of the two dimensional set named Cantor dust is $\mathcal{H}^s(\mathcal{C}^2) = 2^{(s/2)}$, where $s = \log 4/\log(1/\lambda)$. After this, we define a sequence introduced by Jia, which decreases and converges to the Hausdorff measure of any self-similar set satisfying the open set condition.

In the end, we move onto some practical results of the problem. We use the sequence of Jia to get an upper bound on the Hausdorff measure of the Sierpinski triangle, then we give the algorithm of Móra, which uses the same sequence to give the best lower bound known so far, namely we show that:

$$0.77 \leq \mathcal{H}^s(\mathcal{S}) \leq 0.81916$$


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