Algebraic differences of Mandelbrot percolation Cantor sets

ABSTRACT

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In this thesis we study dynamically defined random sets. More precisely, we investigate the family of the so-called Mandelbrot percolation. We review the most important geometric measure theoretical properties of Mandelbrot percolation Cantor sets. Then we turn to the question whether the algebraic difference of independent copies of such Cantor sets has non-empty interior. More precisely, we define the Mandelbrot percolation random Cantor set (in the simpliest case) as follows: we divide the unit square in $\mathbb{R}^d$ into M^d congruent cubs of size $1/M$. Another parameter of the construction is a probability $p \in (0,1)$. With this probability p we retain each of the M^d cubs of size $1/M$ independently. In those cubes which were retained we repeat the process independently. In the discarded cubes we do not do anything. The Mandelbrot percolation Cantor set is what remains after infinitely many steps.

It was pointed out by Chayes, Chayes, Durrett that on the plane ($d = 2$) if the retention probability is high enough then the Mandelbrot percolation Cantor sets connects the opposite walls of the unit square with positive probability. We describe some other interesting geometric measure theoretical properties of the Mandebrot percolation Cantor sets.