On the sizes of components in the supercritical village model

Abstract of Master Thesis

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The finite village model $Q_{n,m}(\mu)$ is a random graph model with community structure, which we think of as villages with n inhabitants arranged along a cycle of length m. Any pair of individuals who are at most one village away may be in contact with probability $p_n := \frac{\mu}{3n+1}$, independent from any other pair. Formally, the underlying deterministic graph is the strong product of the cycle C_m and the complete graph K_n, then we apply independent Bernoulli percolation with edge retention probability p_n. The parameter μ is the asymptotic number of neighbors as $n \to \infty$. The supercritical regime of the model is $\mu > 1$.

It is a natural question which of its factors the village model will resemble. We chose to study the aspect of percolation component sizes, that is, whether a so-called giant component of size linear in nm exists, which differentiates percolation on the complete graph and the cycle. We believe that the village model displays a bit of both behavior, depending on the relation between m and n as they tend to infinity. If the cycle is 'short enough' compared to the size of the villages, a giant component emerges, while a 'long enough' cycle leads to cycle-like behavior of the village model as well. We believe this threshold is of magnitude $m \sim \mathrm{e}^{Cn}$.

To establish this, we introduce the infinite village model $Q_{n,\infty}(\mu)$, defined by replacing C_m with the integer line $\mathbb{Z}$, and study the sizes of components intersecting the origin. Denoting the rightmost village with an individual connected to the origin by M_n, we prove exponential upper and lower bounds for $\mathbb{E}[M_n]$. In the barely supercritical regime $\mu = 1 + \varepsilon$, with ε independent of n, we give a finer asymptotics on $\frac{1}{n} \log \mathbb{E}[M_n]$ in terms of powers of ε.