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Longstaff-Schwartz American Option Pricing Algorithm

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Abstract

In this B.Sc. thesis, I present the American option pricing algorithm developed by Longstaff and Schwartz in 2001. The aim of this work was to understand the theoretical background of the method, and present it to readers with a B.Sc. level mathematical background. I implemented the method in Wolfram Mathematica and examined its convergence on Bermudan options, as well as its behaviour when changing the different parameters, such as basis functions, exercise dates and the number of paths used for simulation.

In Chapter 1, I present most of the fundamental definitions, concepts and theorems related to American option pricing. The chapter aims at giving sufficient background to understand the objective of the LSM method. These include various option definitions, a brief overview of the Black-Scholes model, stopping times, the risk-neutral measure and the Snell envelope for pricing American options. The chapter also introduces the geometric Brownian motion as a way of modeling stock prices.

In Chapter 2, I introduce the Longstaff-Schwartz algorithm somewhat generally, but mostly focus on American put options with a single underlying Markovian stock price process, such as the geometric Brownian motion. The algorithm is demonstrated on a simple example, followed by a section on convergence theory where I examine various theoretical results on the convergence of the algorithm to the true option price.

In Chapter 3, I illustrate my implementation of the algorithm in Wolfram Mathematica, and present my results for various tests. I benchmarked my implementation results to finite difference values, and then tested convergence while increasing the amount of paths, exercise dates and basis functions. Finally, I examined the cross-sectional regressions and the generated stopping rules.