

ABSTRACT

In this thesis, we present a selection of results pertaining to the SABR (Stochastic Alpha Beta Rho) model, that is an industry standard for European-style payoffs. The models received quite a lot of attention in recent years. Practitioners use the model due to its flexibility and behavior being in line with observed market movements, hence providing reliable hedges. The model formulates the dynamics of the forward rate the following way, after a suitable measure change to the T -forward measure:

$$\begin{aligned}dS_t &= \alpha_t S_t^\beta dW_t & S_0 &= S \\d\alpha_t &= \nu \alpha_t dB_t & \alpha_0 &= \alpha \\ \langle dW_t, dB_t \rangle &= \rho dt\end{aligned}$$

After defining the mathematical and financial framework. we introduce the SABR model, detail the proof of the call price in case of $\rho = 0$ and the related mapping following the work of Antonov and Spector. Then, we present a short summary of the original result of Hagan, corrected by Obloj. To wrap up the theoretical section of the thesis, we give a short oversight of the work of Paulot, who used a differential-geometry based approach to arrive to an approximation. Finally, we present some results on the calibration of the model and numerical comparison of the aforementioned formulas. The application of the approximations is not without drawback, though, in some cases, the higher-order terms result in negative implied density, and therefore showing signs of arbitrage.