Limit theorems for directed polymers with boundary perturbations

Zsófia Talyigás

The Kardar-Parisi-Zhang (KPZ) equation is a nonlinear stochastic partial differential equation which describes the temporal change of a random height function depending on time and space. A growth model is considered to be in the KPZ universality class if its long time behavior is similar to the behavior of the KPZ equation itself. The analysis of the KPZ equation and universality class is a current research topic in modern probability theory. Directed random polymers are strongly related.

In this work a semi-discrete random polymer model with log-gamma boundary sources will be examined first. Under appropriate scaling the partition function of the model converges to the solution of the stochastic heat equation (which is also the partition function of the continuum polymer), or equivalently, the free energy of the same model tends to the Hopf-Cole solution to the KPZ equation (which is also the free energy of the continuum polymer). To make clear the initial conditions is not trivial and it is a task of our work.

Our first main result is a limit theorem for the free energy of the semi-discrete polymer model. The idea of the proof is to find the limit of a particular Fredholm determinant with the help of a critical point analysis. This convergence implies the limit theorem.

Our second important result is a limit theorem for the free energy of the continuum polymer model. A Fredholm determinant formula for the Laplace transform of the continuous partition function is needed for the proof, and it is also an important result of this work.

The gained limiting distribution and the free energy fluctuation are in accordance with the KPZ universality conjecture in both cases.