Abstract of the master's thesis titled Graphical models and some related algorithms

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In this work we give an introduction into the theory of probabilistic graphical models and show some related algorithms. The purpose of graphical models is to provide a clearer, computationally more efficient way to look at multidimensional probability spaces. The aim of these graph-based representations is to give us a picture about the dependencies that hold between the variables.

After an introductory part we show examples of graphical models and point out some connections between the different types. Two of the most commonly used models are the Bayesian networks and the Markov random fields. We start with Bayesian networks which are widely used models in case of directed graph representations. Then we continue with Markov random fields, which are undirected models. We describe the log-linear models too, which are not necessary graphical models, but they have their statistical and information theoretical importance. We also give a brief introduction into multidimensional normal distributions and the Gaussian graphical models, which are built on these distributions. We stick mainly to discrete variables, however supplements about these continuous models, the Gaussian models, are also included.

We thoroughly investigate a useful graph property, the chordality. This is equivalent to the graph being decomposable, which opens the door to many applications. We examine equivalences of the decomposable graph property. Since this is a desired feature of the underlying graph, we show commonly used methods to identify these graphs, or modify a given graph in order to get a chordal representation.

At the same place we also introduce the junction tree structure, which will be used during the detailed description of the junction tree algorithm. This is a specific tree-structure of the cliques of the graph. This algorithm applies the belief propagation method on the junction tree structure of the graph. The goal of this algorithm is to find marginal distributions for the variables in a graphical model, possibly after absorbing some evidences. This method makes the marginalization computationally more effective, but it can only work if the probability distribution is a Gibbs-distribution with respect to the given graph (this feature will be introduced in the context of Markov random fields), and the graph representation of the probability space is triangulated (chordal), possibly after some modifications. At the end we show a detailed application based on a stylized example.

Summarizing, our main purpose is a thorough understanding of the theoretical background of the graphical models, and pointing out some connections between them by the examination of decomposable models.