Abstract

An Interacting Particle System: the Contact Process

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The contact process is an interacting particle system. It is a continuous time Markov process with state space $\{0,1\}^S$, where S is a finite or countably infinite graph. The process is usually interpreted as a model for the spread of an infection: if the state of the process at a given time is η, then a site $x \in S$ is "infected" if $\eta(x) = 1$ and healthy if $\eta(x) = 0$. Infected sites become healthy at a constant rate 1, while healthy sites become infected at a rate equal to λ times the number of infected neighbors, where the infection rate λ is a positive real parameter.

Let us consider the probability of the event that the infection survives forever if we start from one single infected site. This probability is a monotone non-decreasing function of λ. One defines the critical threshold λ_c as the supremum of those values of λ for which the survival probability is equal to zero. It is known that if $S = \mathbb{Z}^d$ then $0 < \lambda_c < 1$. The question naturally arises: can the infection survive forever if $\lambda = \lambda_c$? This question has been open for many years, and was settled in *Carol Bezuidenhout and Geoffrey Grimmett: The critical contact process dies out*, where it was shown that the critical contact process on $\mathbb{Z}^d$ does die out with probability one. This thesis contains a detailed proof of this result. The main idea of the proof is to show that there is a condition which (a) is equivalent to the survival of the contact process, but (b) only depends on the behaviour of the contact process in a finite space-time box.

The exact value of λ_c is currently not known. Our thesis contains the results of our computer simulations aimed at the approximation of the critical threshold λ_c of the contact process on $\mathbb{Z}^2$.

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