Excited random walks

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The goals of the thesis

We consider excited random walks, a class of self-interacting random walks in deterministic or random environment on the d-dimensional integer lattice. At the beginning of the thesis, we introduce the set of environments. The environment is chosen randomly, according to a measure $\mathbb{P}$ on the appropriate measurable space. We introduce the most common assumptions about $\mathbb{P}$, and some special models. One of the goals of the paper is to find the right assumptions, under which ERWs have properties similar to the ones, that simple random walks possess. These are the range of the walk (being finite or infinite), recurrence, transience, a law of large numbers and ballisticity. An interesting connection with branching processes with migration is also observed.

The structure of the thesis

The thesis consist of 5 chapters; Introduction, Range of ERWs, Recurrence and transience of ERWs, Strong law of large numbers and Ballisticity.

Introduction

In this chapter, we describe a general model for ERWs. We introduce assumptions on $\mathbb{P}$, when the cookie stack are i.i.d. (IID), when the cookie stack are stationary and ergodic (SE), ellipticity conditions (WEL), (EL), (UEL), and some common models, notably (POS_ℓ) and (BD).

Range of ERWs

In this chapter we give necessary and sufficient criteria for the range to be either finite or infinite. When discussing one dimensional walks, we assume (SE) and (EL). For multidimensional walks, we assume (IID) and (EL).

Recurrence and transience of ERWs

In this chapter we introduce the parameter δ. For the one dimensional ERW, under (SE) and (EL), a 0-1 law is given, about its recurrence and transience. We also mention, that the recurrence of the one dimensional (BD) under (SE) and (EL) is a function of δ. For higher dimensions we introduce directional transience and recurrence. A Kalikow-type 0-1 law is also explained.

Strong law of large numbers

In this chapter we explain what it means, that an ERW satisfies the strong law of large numbers. We explain, that in the one dimensional case, under (SE) and (POS_1), the ERW satisfies a strong law of large numbers with a deterministic speed. For $d \geq 1$, we introduce the regenerating structure, and using that we introduce a directional law of large numbers.

Ballisticity

In this chapter we define what it means for an ERW to be ballistic. In the case of the one dimensional (BD), we explain that under (IID) and (WEL), ballisticity is a function of δ. When proving this, we describe a link between ERWs and branching processes with migration.