Random walk on projective circle

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In my thesis we try to find measures on the projective circle, i.e. the set of directions on the real plain, which in a way have some stationarity properties to a given random walk.

At first we prove an Ergodic Theorem, the corollary of which will be quite useful later.

Then in the first chapter we introduce the upper Lyapunov exponent, this value associated with a particular random walk will prove itself to be handy when we are trying to figure out if this stationary distribution exists. After that we define the basics of an additive cocycle and the already mentioned stationarity in form of invariance of an other measure. Then we show that there is an easier way to calculate the Lyapunov exponent.

In chapter two we start to talk about matrices of order two. Firstly we define the projective circle, then we state and prove two lemmas. The first one talks about the convergence of a measure under the affect of the random walk, and the second one shows some properties of the integral of an additive cocycle wrt our measures. The second lemma will be the key to understand the connection between the Lyapunov exponent and the invariant measure. After that we prove two theorems about the contraction of vectors under any realization of the random walk. And we arrive at the main theorem, which collects the results of the previous statements, and gives loose conditions for the existence of the unique, continuous, stationary measure. At the end we state sufficient conditions for the previous theorem.

In the last chapter we show a simple example, for which we find the stationary measures and calculate the upper Lyapunov exponent.