

Abstract

This paper is divided into four main parts. The first part is covered in Chapter 1 and 2. First, the task is introduced with the motivation and goal of this paper and in Chapter 2 the mathematical concepts used throughout this paper are briefly summarized. After introducing the Euler–Lagrange equation, Routh’s reduction is presented and demonstrated on a simple cyclic mechanical system. Basic balancing pendulum types are also reviewed with a short introduction on how to stabilize them around their unstable equilibrium.

The second part is Chapter 3, where we revisit Routh’s reduction from the previous chapter and take an unusual step to include the external force acting on the system. After discussing this idea, we generalize this method to cyclic systems with arbitrary number of generalized coordinates. The main result of this thesis is that this method will also apply to systems that are actuated at their cyclic coordinates. From this general formula we will see that in case of one essential coordinate the result can be further simplified.

The third part includes Chapter 4 and 5. In Chapter 4 we introduce a system called the Furuta pendulum, for which the development of the mathematical model is also detailed. Afterwards, we will use our newly found result to eliminate the cyclic coordinate from the equations of motion of the Furuta pendulum, while still capturing the full dynamics of the forced system. We conclude that ”traditional” elimination procedure leads to the same result and then we analyze the resulting equation for the essential motion. Afterwards, in Chapter 5 we compare the numerical behavior of the original system of equations and the equation of the essential motion for both the controlled and uncontrolled cases.

The thesis is closed by Chapter 6 with a summary of the results, conclusions and an outlook on future work.