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Pareto-optimality with fixed coalition size

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This thesis is about coalitions, more precisely Pareto optimal coalitions of fixed size. The problem is partitioning players into pre-fixed size groups. Each player has preferences over the other ones. We have studied several settings, which differ in the size of coalition, in the preferences, in the way calculating the value of a coalition.

We dealt with Pareto Optimal assignments. We say that an assignment is Pareto optimal if there is no other assignment which is strictly better for one without being worse for the others. Our settings involved n players who need to be partitioned into coalitions, i.e. every player has to be assigned to a room. We restricted our settings to having fixed size rooms, i.e. 3-persons room, strict and complete preference lists. We verifyed whether a given assignment is PO or not. We presented two proof about verifying. The difference between them was the following: in the first case the worst roommate defined the difference between rooms and in the second, the best roommate did the job, so the player can differ two coalitions based on their worst or best assigned roommate. In the case when the worst counts, we gave a mapping to the Triangle Cover problem, which is a known NP-complete problem. This means that verifying PO in this settings is NP-complete. In the case when the best roommate counts, we also gave a mapping to a known NP-complete problem, called Directed Triangle Cover.
We showed that finding Pareto optimal assignment can be easy but can be hopeless too. We used the fact that PO is guaranteed to exists when the preference lists are complete. We showed three easy cases when we can give an algorithm to solve the problem. These cases had strict, complete lists and in the first the best roommate counts with 3-person rooms, in the second worst counts with 3-person rooms and in the last one, worst counts with r_i-person rooms. We concluded, variants of serial dictatorship works for these cases, so we could find the required assingments. In the hopeless cases, we could give a mapping to NP-complete problems again.
As conclusion, we can say that verifying Pareto Optimality is hard, moreover it is NP-complete. Findig Pareto optimal assignment can be easy with serial dictatorship, but in some cases it is hopeless to find it in polynomial time.

Finally, we stated two algorithms and reviewed the terminology before them. We started with perfect partition, which is a partition where everybody gets the most preferred coalitions. We know that every perfect partion is Pareto optimal, so if we find a perfect partition then we get the PO one too that is why perfect partitions are important for us. We gave an algorithm, called PerfectPartition which helps us to find a perfect partition.
Then, we entered the Preference Refinement Algorithm and reviewed the needed concepts, such as refinements, coarsening and we used the PefectPartition algorithm too. We stated a theorem which characterizes Pareto optimal partitions for a preference list R as those that are perfect for coarsenings R' of R such that for no preference list R'' with $R < R'' \leq R'$ perfect partitions exist. We discussed the PRA algorithm and stated a theorem which claimed that PRA returns an individually rational and Pareto optimal partition for every hedonic games. We studied some types of PRA and talked about computational efficiency. In the end, we also compared PRA with Serial Dictatorship via an example.
As a conclusion, we can say that PRA works well, further it works better than SD sometimes.