Abstract

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The aim of this thesis is to get acquainted with the thermodynamical formalism of the dimension theory of fractals generated by iterated function systems, gather the basic tools to deal with this subject and investigate the basic properties and relationships between them. In particular, this text is concerned with measure theoretic and topological entropy, topological pressure of transformations, the existence of Gibbs-measure, Hausdorff-measure and Hausdorff-dimension, iterated function systems and so called Lyapunov exponent. The most important theorems in this text will be given proofs, all the rest will be given references to the proofs.

At the beginning of the first section the symbolic space and related notions will be discussed, followed by the very important concept of measure theoretic- and topological entropy and the connection between them. The theorems by Kolmogorov and Sinai will provide a major method to calculate these quantities. At the end of the section, a sufficiently detailed discussion on the existence of Gibbs-measure will be given along with a sketch of the proof. This measure has an important relation to the topological pressure, which is called the variational principle. Particular interest will be taken in Ruelle's Perron–Frobenius Theorem, it will be proved with the help of a long sequence of lemmas. This theorem provides the main tool in the proof of the existence of Gibbs-measure.

In the second section a popular measure in the study of fractals, namely the Hausdorff-measure will be defined, followed by its most important properties and an example of the calculation of the Hausdorff-dimension of a fractal set called Cantor Dust Set. At the end of the section iterated function systems are investigated briefly and an example on an application of the topological pressure, relevant to iterated function systems, is given.

The last section gives a brief introduction on the Lyapunov-exponent, and the important theorem of Oseledec.