Self-organized criticality of

dynamic random graph models

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In the thesis we investigate the growth of clusters in the Erdős-Rényi forest fire model and the dynamical Erdős-Rényi random graph model. These models are random graphs that evolve in continuous time in a Markovian fashion. In the forest fire model connected clusters "burn down" (i.e. every edge of the cluster is deleted) due to a Poisson point process of lightnings. Each edge will be able to appear according to another time-homogeneous Poisson point process. In the dynamical Erdős-Rényi random graph model there will be no so-called lightnings. We talk about the connection of the proportion of mass in clusters of a specific size in the Erdős-Rényi model and the Borel distribution. We describe the cluster growth in the two models in the first chapter of the thesis, however the proofs of the claims we make will appear later on.

In the second chapter of the thesis we prove that Flory's equations describe dynamics of the cluster growth of the dynamical Erdős-Rényi random graph model by showing that the Borel distribution solves Flory's equations. We make a note of Cayley's formula (and provide a proof for it), because that will be needed in the proof. We talk about the phase transition of the Borel distribution and its connection to the Galton-Watson branching process with Poisson offspring distribution.

In the third chapter we provide a stationary solution to an infinite system of differential equations that describe the forest fire model. For this, we have to make note of the Catalan numbers and describe some of their properties. We give an explicit formula for Catalan numbers and provide two proofs for that formula, in the first we use some properties of Dyck paths, in the second we use the generating function of the Catalan numbers.

We have done computer simulations on the random graph models to confirm some of the claims we make in the thesis. We talk about how we represented the continous-time models as discrete-time models and how the simulations confirmed some earlier conjectures. We also simulated two other models, the cycle induced forest fire model and the cycle deleting model. We discuss the results of the simulation we did on these models and talk about how they compare to the forest fire model.