Mandelbrot percolations with inhomogeneous probabilities

Vilma Orgoványi
Supervisor: Prof. Károly Simon

In this thesis we consider geometric measure theoretical properties of a family of random fractals, namely the Mandelbrot percolation fractals in two dimension. The construction of the Mandelbrot percolation fractal consist of infinite iteration of the following steps. Our initial set is the unit square $[0,1]^2$. The squares which we have after the n-th iteration we subdivide into smaller squares of the same size, and the smaller squares are retained or discarded. Retaining or discarding different squares are independent random events. The percolation fractal consist of those points which we have never discarded. We distinguish two cases the homogeneous one, when we retain the squares with the same probability, and the inhomogeneous one, when the probabilities are not the same. We define conditions which, if satisfied, guarantees almost sure non-empty interior for the orthogonal projection of the percolation fractal to all directions. We show that in the homogeneous case, if the Hausdorff dimension of the set is greater than one, the interior of the projection is not empty with probability one, we also show that this does not hold in general in the inhomogeneous case. We define a random measure for all level n approximation set, and show that this sequence of measures converges in a weak* sense to a measure supported on the Mandelbrot percolation fractal. Finally we present the proof of the result of Michał Rams and Yuval Peres, that is, in the homogeneous case, if the Hausdorff dimension is greater than one, then with probability one, all orthogonal projection of the above mentioned measure are absolutely continuous and, except for the horizontal and vertical projection, have Hölder continuous density.