Analysis of the college admission problem with different tie-breaking policies

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Matching problems are fundamental in combinatorial optimization. The *higher education admission*, the *public school choice* and *kidney exchanges* are basic examples of how matching mechanisms are applied nowadays in economics. In this thesis, we study the so-called *college admission problem*. In 1962, Gale and Shapley introduced an algorithm which solves this problem in the case where preferences are strict, and Alvin E. Roth has further studied the problem in several papers. In 2012, they have received the Nobel Memorial Prize in Economic Sciences "for the theory of stable allocations and the practice of market design".

In this thesis, we introduce special features of the college admission problem such as lower and common quotas, paired applications and ties in preferences. We also discuss that some of them make the problem NP-complete. In Hungary, the admission to higher education institutions has been centralized since 1985, and the matching scheme has the previously mentioned special features; thus in practice, it is reasonable to use different heuristics.

We analyze the college admission problem, particularly when only ties are allowed from the above conditions. In this case, the original Gale-Shapley Algorithm can be extended to handle ties. There are different tie-breaking policies worldwide, and we examine three exceptional cases. In Hungary and Chile, an equal treatment policy is applied, i.e. a group of students with the same scores are all handled the same way. In Hungary, no quota violation is allowed, but in Chile, the quota must be adjusted if the particular college does not have enough seats to assign a whole group of students with equal scores. In Turkey, no equal treatment policy is used; moreover, ties in scores are broken by additional criteria, i.e. the algorithm proceeds as the original Gale-Shapley Algorithm.

The main result of this thesis is that we implemented and tested the six main algorithms which can solve the college admission problem with ties. The algorithms are applied to the real Hungarian higher education admission data obtained in 2008. The bases of the algorithms are the applicant- and college-oriented Gale-Shapley Algorithms, and we implemented both of them with the Hungarian, the Chilean and the Turkish tie-breaking policies separately. During our work, we checked whether our solutions are compatible with the previous theoretical results. Moreover, we compared our solutions to each other to see how changes in the current policy would affect the results from the perspective of the students.