Discrete Maximum Principle for Higher Order FEM for Elliptic PDEs

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Abstract

The finite element method (FEM) is one of the most common numerical methods for solving elliptic partial differential equations (PDEs). An important measure of the reliability of implementation on a given problem is whether it preserves the qualitative properties of the original model. Such basic qualitative properties are non-negativity of the solution and maximum principles. For linear elements the conditions of discrete maximum principles are well-known for various elliptic problems. However, for higher order elements effective results were obtained only for one-dimensional problems. The aim of this thesis is to examine the discrete maximum principles on two-dimensional problems. The basic idea is to use Bernstein-Bézier polynomials. These are non-negative and fulfill the partition of unity property. Firstly, we define the relation between the matrix maximum principles and the discrete maximum principles, then examine this relation for various angular conditions of the grid. Secondly, we implement the finite element method with Bernstein-Bézier polynomials, and in test problems we determine the relation between the data, the grid parameter and the fulfillment of the discrete maximum principle.