Hofbauer Pressure for piecewise monotonic invertable dynamical systems

Körtvélyesi Viktor

The goals of the thesis

In most cases, when we study dynamical systems, we are given a continuous self map of a compact set, and we would like to understand the structure of the orbits of a point. However, in this thesis the compact set is the $[0, 1]$, but the theorems and definitions will be declare with an arbitrary $M \subset \mathbb{R}$ compact set. Furthermore the self map of M or $[0, 1]$ is not necessary continuous, just piecewise continuous. We focus on the cases when there is no finite Markov Partition. This is a difficult situation, because the existence of finite partition makes it possible, to describe the dynamics with a subshift of finite time. Franz Hofbauer in the 1980s introduced a technics, which we can use for the interval map, when there is no finite Markov Partition. This is the method of Markov Diagrams, which are basically countably infinite directed graphs. Using this powerful method, one can get better understanding of dynamics, of piecewise monotonic and expanding self maps of the interval. Piecewise monotonic dynamical systems can describe a completely uncertain behavior. They are deterministic systems, but exhibit feature that we experience when we study random systems. We call this uncertain behavior, chaos.

The structure of the thesis

The thesis consist of 4 chapters: Introduction, Markov Diagram, Topological Entropy and Topological Pressure.

Introduction

In this chapter, we describe the basic definitions and we visualize some properties and definitions for a cleaner view of the topic. We introduce the field of chaotic dynamical systems, and we define the piecewise property of a continuous self map.

Markov Diagram

In this chapter we give a simple and detailed introduc- tion to Markov Diagrams, accomponed by number of examples. This construction implies, that the Markov Diagram is a directed graph, such that the verticies will be the successors. The main part of this Chapter consists of few number of theorems and lemmas, related to the Markov Diagram.

Topological Entropy

In this chapter we introduce the topological entropy based on the definition given by Misiurewicz and Szlenk. In further we give a method how to define the topological entropy fo piecewise continuous self maps with discontinuity points. Furthermore this chapter gives a simple method with the Markov Diagram to define the topological entropy without a generator partition.

Topological Pressure

In this chapter we define the topological pressure for piecewise continuous self maps on the interval. We will investigate the connection between topological entropy and topological pressure.