Abstract of the M.Sc. Thesis

Mixing Time of Critical Ising Glauber Dynamics

Author: Gergely Dániel Lukáts
Supervisor: Dr. Gábor Pete

The main focus of our study is the Ising model, which was introduced by Ernst Ising in his doctoral thesis as an attempt to model ferromagnetism, after suggestion from his supervisor, Willhelm Lenz in 1920. The model is a natural site (vertex) percolation model, but with correlations introduced between the sites. It is a family of probability distributions interpreted on a graph (e.g. the square lattice, torus, or the complete graph, called the Curie–Weiss model), where each site can have a *spin value* of either $+1$ or -1, which represent the magnetic dipole moments of individual atoms.

Above a threshold *critical temperature*, ferromagnetic materials undergo a *phase transition* and spontaneously lose their magnetisation. The Ising model has an exponentially large sample space, and is generally simulated using a Markov chain known as the *Glauber dynamics* which converges to the *Gibbs distribution* of the Ising model. The order of steps needed for the dynamics to be sufficiently close to the Gibbs measure is known as the mixing time, which is an intensively studied topic, with many unsolved problems that are only conjectured through simulation results.

In our study, we examine the deep mathematical background of the Ising model, coupling, mixing and relaxation times. We do a thorough analysis of recent results for the Ising model on the complete graph. We provide a generalised modification of the proof of its critical mixing time (by Ding et al.), and use intuition from the calculation techniques to provide conjectures for the Erdős–Rényi random graph and the square lattice. For our research, we created countless computation-heavy simulations to better understand and explain the dynamics and theory of these models.

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Budapest University of Technology and Economics
Faculty of Natural Sciences