Variational inequalities

Abstract

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We describe the whole process of modelling and solving the obstacle problem with the help of the theory of partial differential equations, optimisation and numerical methods.

In the second section we derive the obstacle problem through physical laws and reformulate it in various ways. As a constrained differential equation, as a free-boundary problem, as a functional minimisation problem and as a variational inequality. Motivated by this special type of inequality, we summarize the functional analysis background needed in the third section. Then in the fourth section, formally introduce the elliptic variational inequalities of the first kind in general. We prove existence and uniqueness of their solution, link them to equivalent functional minimisation problems and prove the equivalence.

The fifth section describes the discretization of the domain of the physical problem and the underlying function space of the solution with proof of convergence. The relaxation with projection method for solving the discretized problem is discussed in the sixth section. Finally, the seventh section concludes the thesis with the numerical solution of the obstacle problem in one and two dimensions.