The Riemann–Roch Theorem and Its Applications

Ferenc Lukács

Abstract

Algebraic geometry is, and has been for a long time, an important and actively developing area of pure mathematics. Because of this, the volume of mathematical thought this field encompasses is enormous. In this thesis we only attempt to cover a tiny fraction of it.

The main focus of this thesis, the Riemann–Roch theorem, originally proven for Riemann surfaces in 1865, is an important result in both complex analysis and algebraic geometry. Owing to the fact that it can be generalized in a number of different ways and relates algebraic or complex analytic information to purely topological invariants, the *genus* and the *degree*, the theorem has many far-reaching applications, of which this thesis contains a selected few.

We attempt to give a proof of the Riemann–Roch theorem for curves and for surfaces with as little preliminaries as possible, while trying to keep the thesis as self-contained as possible. Along with each of these we showcase a variety of applications from the theory of curves and surfaces. Among these applications is the discussion of the canonical embedding of curves into projective space, the Nakai-Moishezon criterion for ample divisors and Castelnuovo's theorem that characterizes contractible curves on a surface.